Advantages of adaptive and general strategies for discrimination of unitary channels beyond group-theoretical methods

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QUICK SUMMARY

For the task of minimum-error channel discrimination of unitary channels, several previous results proved the optimality of parallel strategies in some scenarios. Here, we show that sequential (i.e. adaptive) strategies are in fact advantageous in most scenarios that were not previously studied (higher number of copies and candidates, non-uniform prior distributions, discrimination of sets of unitaries that do not form a group). We also show that strategies involving indefinite causal order outperform even sequential ones in these scenarios and derive an ultimate upper bound for the discrimination of unitary channels.

THE PROBLEM

Alice is given an unknown unitary channel, drawn with a certain probability from a known ensemble of a finite number of unitary channels. Being allowed to use a finite number of copies (uses) of this unitary, her task is to determine which unitary she received. This problem is equivalent to Alice extracting the 'classical information' which is encoded in the 'label' of the unknown unitary channel. Her probability of successfully guessing which unitary she holds will depend on the strategy used to extract information from it. If Alice has access to a single copy, her maximal probability of successful discrimination is given by

$$P := \max_{\rho,\{M_i\}} \sum_{i=1}^{N} p_i \operatorname{Tr}[(U_i \otimes \mathbb{I})\rho(U_i^\dagger \otimes \mathbb{I})\, M_i]. \quad (1)$$

Alice's strategy can be equivalently expressed using a tester, a mathematical object that combines the information of both state and measurement, has a simple mathematical characterization, and allows the maximal probability of successful discrimination to be equivalently expressed as

$$P = \max_{\{T_i\}} \sum_{i=1}^{N} p_i \operatorname{Tr}[|U_i\rangle\langle U_i|\, T_i]. \quad (2)$$

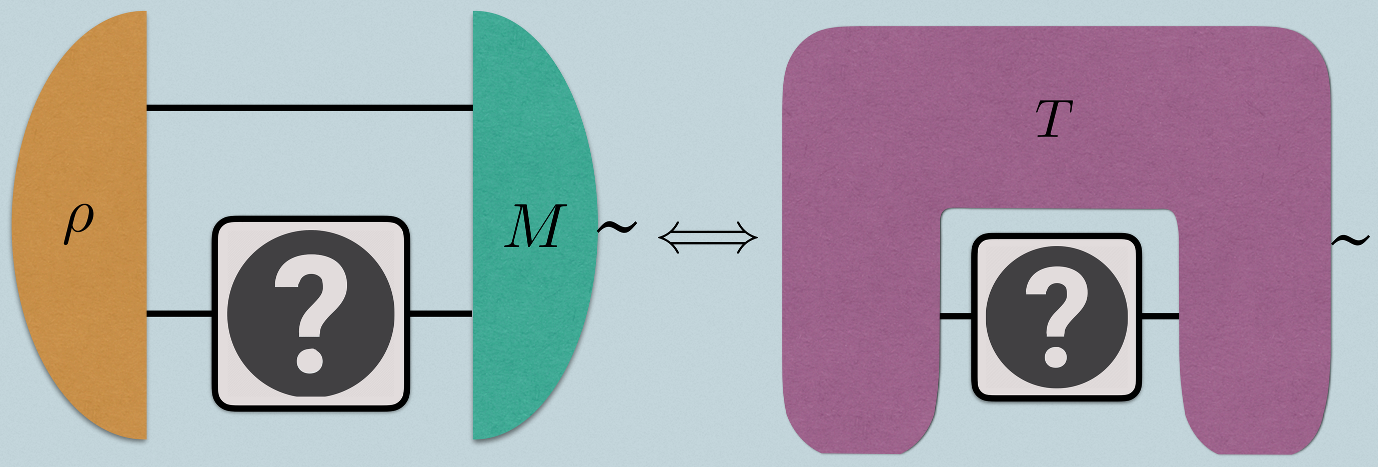

Fig. 01. Alice's strategy of channel discrimination can be equivalently described by the quantum states and measurements she employs (left) or by a *quantum tester* (right), a mathematical object with simple characterisation that behaves as a 'measurement' of quantum channels.

FOR MORE DETAILS

Paper: J. Math. Phys. 63, 042203 (2022)
arXiv:2011.08300 [quant-ph]

See also: PRL 127, 200504 (2021)
arXiv:2011.08300 [quant-ph]

Code: https://github.com/mtcq/

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STRATEGIES

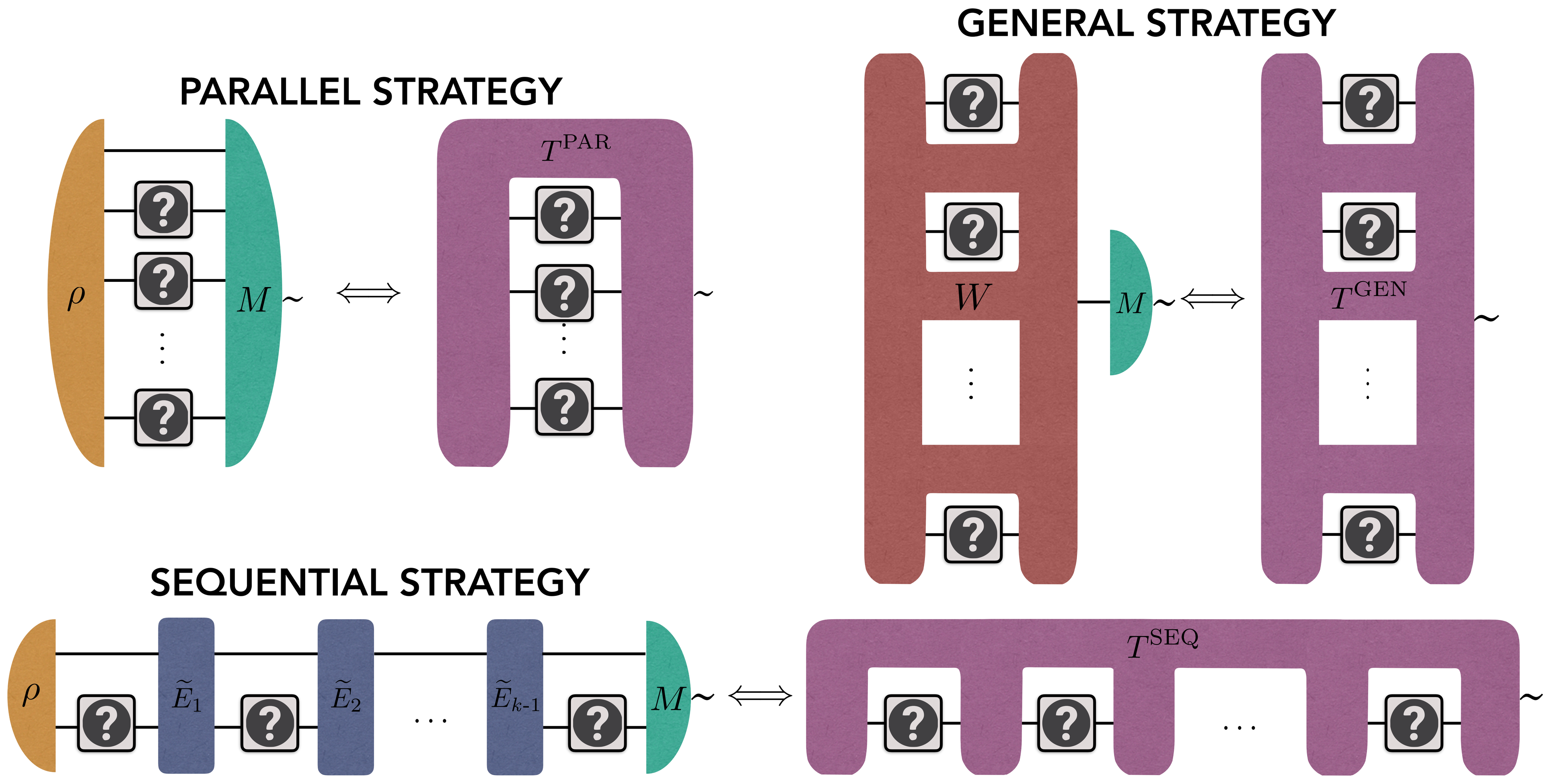


Fig. 02. Schematic representation of different classes of multi-copy strategies for channel discrimination. Each strategy acts on the copies of the unknown channel in a different order: in parallel (top, left), in sequence (bottom), or in an indefinite causal order (top, right).

RESULTS

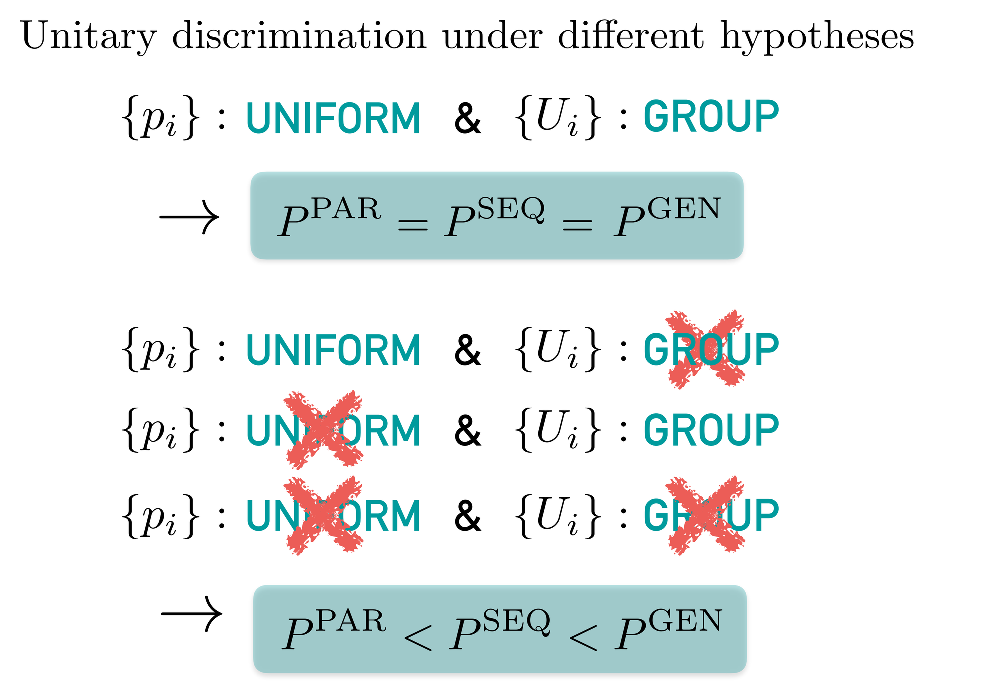
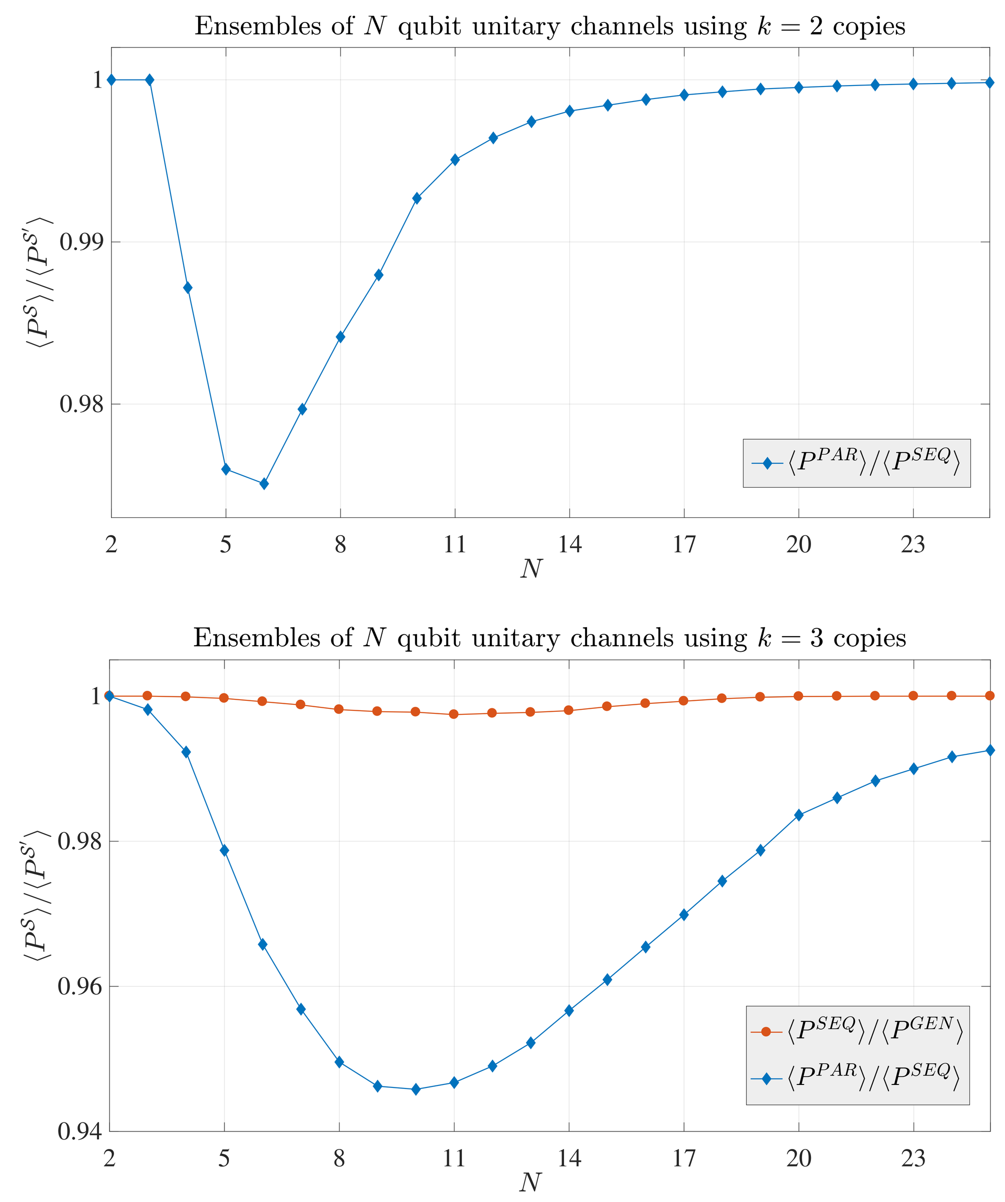


Fig. 03. If the unitary channel ensemble to be discriminated is composed of a uniform prior distribution and a set of unitaries that forms a group, then parallel strategies are optimal even when compared to general strategies. If any of these two hypotheses is abandoned, then there always exist examples of a strict hierarchy of strategies.

Uniformly sampling qubit unitary channels

N	$k=2$	$k=3$
2	$\boldsymbol{P^{\text{PAR}} = P^{\text{SEQ}}} = P^{\text{GEN}}$	$\boldsymbol{P^{\text{PAR}} = P^{\text{SEQ}}} = P^{\text{GEN}}$
3	$P^{\text{PAR}} = P^{\text{SEQ}} = P^{\text{GEN}}$	$P^{\text{PAR}} < P^{\text{SEQ}} = P^{\text{GEN}}$
4	$P^{\text{PAR}} < P^{\text{SEQ}} = P^{\text{GEN}}$	$P^{\text{PAR}} < P^{\text{SEQ}} < P^{\text{GEN}}$
⋮	⋮	⋮
10	$P^{\text{PAR}} < P^{\text{SEQ}} = P^{\text{GEN}}$	$P^{\text{PAR}} < P^{\text{SEQ}} < P^{\text{GEN}}$
⋮	⋮	⋮
25	$P^{\text{PAR}} \approx P^{\text{SEQ}} = P^{\text{GEN}}$	$P^{\text{PAR}} < P^{\text{SEQ}} \approx P^{\text{GEN}}$

Fig. 04. Gaps between different strategies of discrimination using k copies of ensembles of N uniformly distributed qubit unitary channels sampled according to the Haar measure. A strict inequality indicates that examples of ensembles that exhibit such gaps were encountered. An equality indicates that, for all sampled ensembles, no gap was encountered.



$$P^{\text{GEN}} \leq \frac{1}{N}\frac{(k+d^2-1)!}{k!(d^2-1)!} = \frac{1}{N}\gamma(d,k) \quad (3)$$

Upper bound. We derived the above ultimate upper bound for the maximal probability of success for any ensemble composed of N d-dimensional unitary channels and a uniform prior distribution, with k copies, under any possible strategy of channel discrimination.

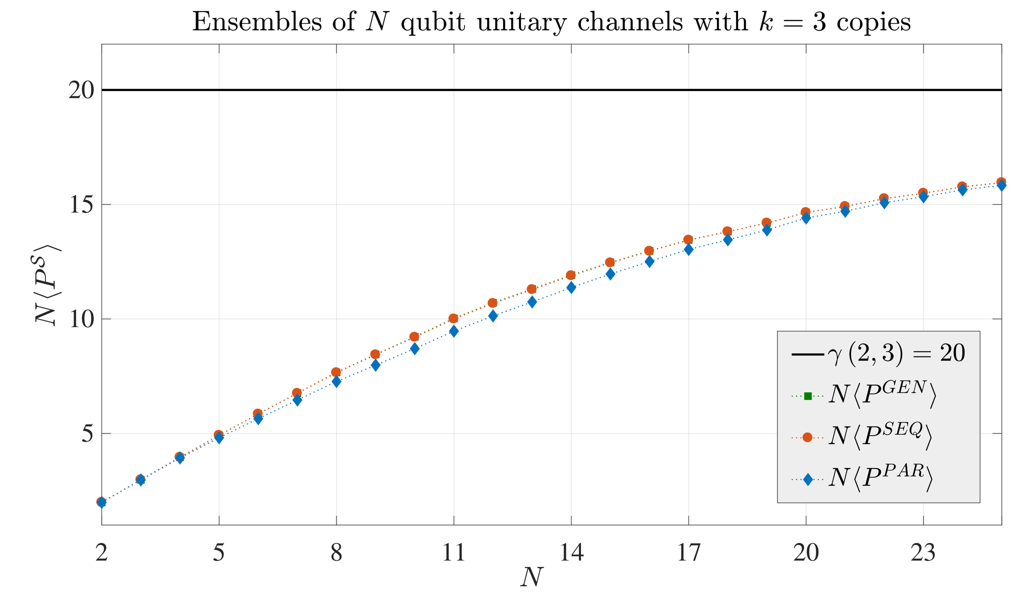


Fig. 05. Left: ratios of the averages of the maximal probability of success for ensembles of N uniformly distributed qubit unitary channels with 2 copies (top) and 3 copies (bottom). Right: upper bound and average of the maximal probability of success for ensembles of N uniformly distributed qubit unitary channels.

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