TWO MEASUREMENTS ARE SUFFICIENT TO CERTIFY

HIGH-DIM ENTANGLEMENT

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IQOQI Vienna

ENTANGLEMENT
AS THE CORNERSTONE
OF QUANTUM COMMUNICATIONS

BUT WHY HIGH-DIM?

GOING BEYOND QUBITS

ENHANCED SECURITY FOR QKD

INCREASED CHANNEL CAPACITY

HIGHER NOISE RESISTANCE

HIGH-DIM ENTANGLEMENT FOR FREE

EFFICIENT

CERTIFICATION

STATE ASSUMPTION-FREE

UNTRUSTED SOURCE

DEVICE DEPENDENT

1) Which state is generated?

Fidelity Bounds

2) How entangled is the state?

Schmidt Number Witness

Measure of entanglement dimensionality:

Schmidt Number

Minimum number of levels needed to represent a state and its correlations in any basis

$$|\Psi\rangle = \sum_{m=0}^{k-1} \lambda_m |mm\rangle$$

$$k_{\max} = d$$ → Local Dimension

Measure of entanglement dimensionality: Schmidt Number

Minimum number of levels needed to represent a state and its correlations in any basis

$$k(\rho) = \inf_{\mathcal{D}(\rho)} \left\{ \max_{|\psi_i\rangle \in \mathcal{D}(\rho)} \{\mathrm{rank}\,(\mathrm{Tr}_B |\psi_i\rangle\langle\psi_i|)\} \right\}$$

$$k_{\max} = d$$ → Local Dimension

Starting Point

$$F(\rho, \Phi) \leq B_k(\Phi)$$

R. Fickler *et al.*, Nat. Commun. **5**, 4502 (2014)

Starting Point

$$|\Phi_{(k')}\rangle = \sum_{m=0}^{k'-1} \lambda_m |mm\rangle$$

$$F\big(\rho_{(k)}, |\Phi_{(k')}\rangle\big) \leq \underbrace{\sum_{m=0}^{k-1} \lambda_m^2}_{B_k(\Phi_{(k')})}$$

R. Fickler *et al.*, Nat. Commun. **5**, 4502 (2014)

TWO MEASUREMENTS

$$\tilde{F}(\rho, \Phi) \leq F(\rho, \Phi) \leq B_k(\Phi)$$

WITHOUT ASSUMPTIONS ON ρ

$$|\Phi\rangle = \sum_{m=0}^{k-1} \lambda_m |mm\rangle$$

$$F(\rho, \Phi) = \sum_m \lambda_m^2 \langle mm|\rho|mm\rangle + \sum_{m,n\neq m} \lambda_m \lambda_n \langle mm|\rho|nn\rangle$$

$$= F_{\text{diag}}(\rho, \Phi) + F_{\text{off-diag}}(\rho, \Phi)$$

$$\geq F_{\text{diag}}(\rho, \Phi) + \tilde{F}_{\text{diag}}(\rho, \Phi) + \tilde{F}_{\text{tilted}}(\rho, \Phi))$$

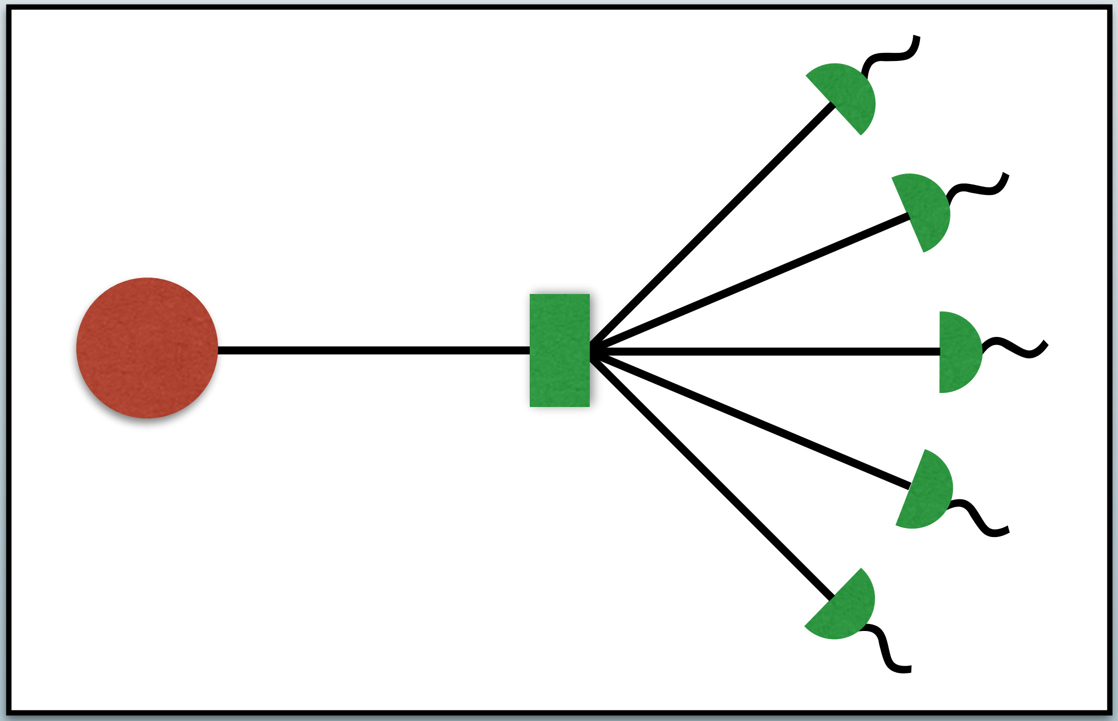
FIRST MEASUREMENT

SECOND MEASUREMENT

$$\geq \tilde{F}(\rho, \Phi)$$

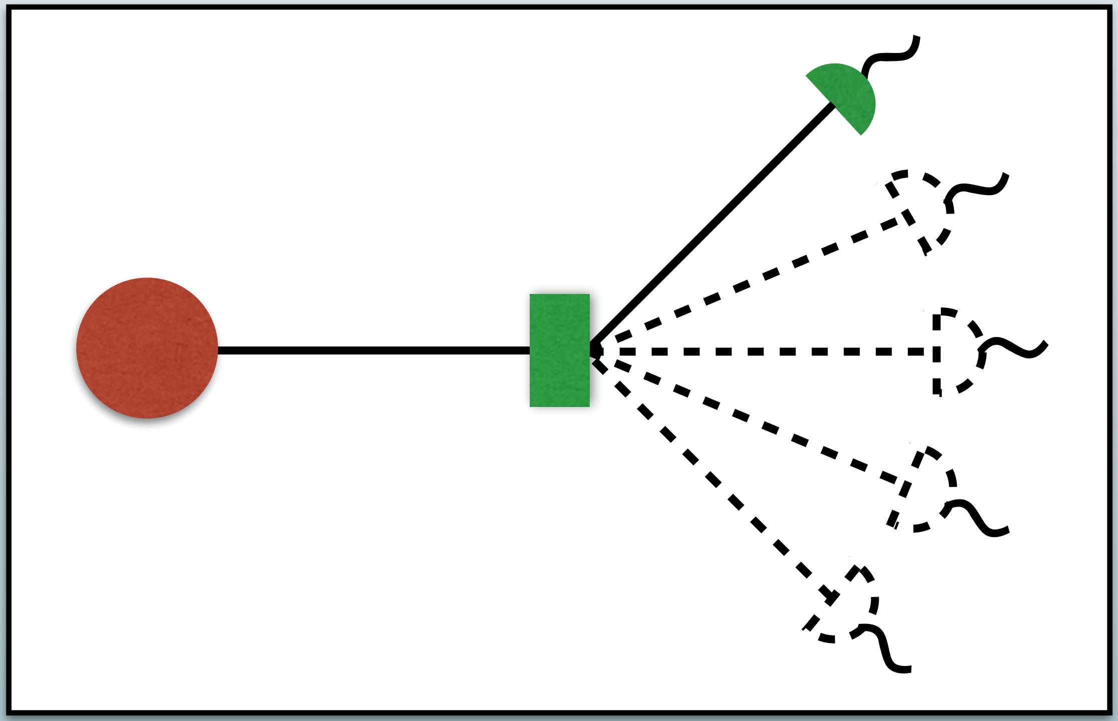
What counts as a measurement?

IMPLEMENTING A d-OUTCOME MEASUREMENT

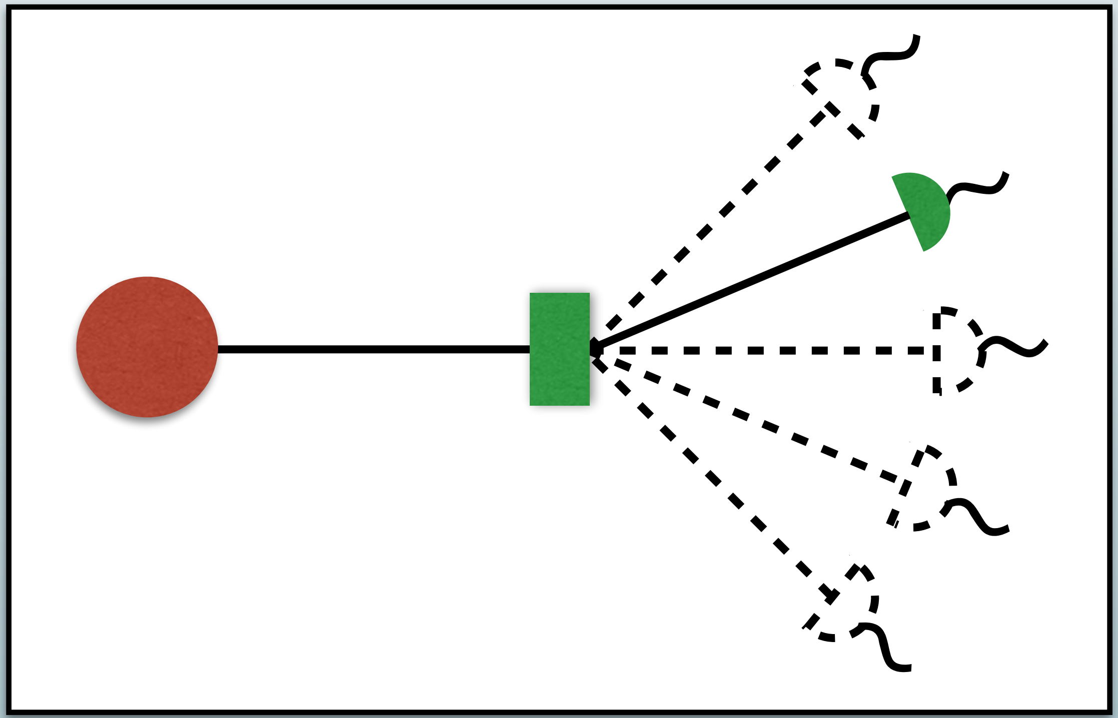


d-outcomes

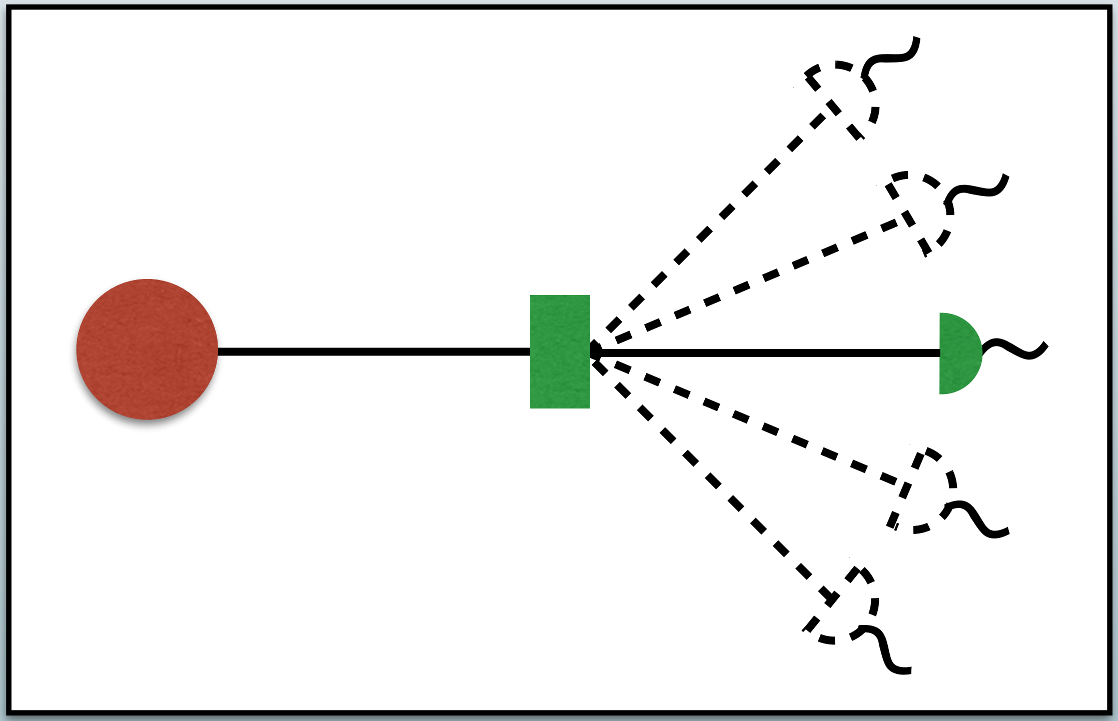
IMPLEMENTING A d-OUTCOME MEASUREMENT



IMPLEMENTING A d-OUTCOME MEASUREMENT

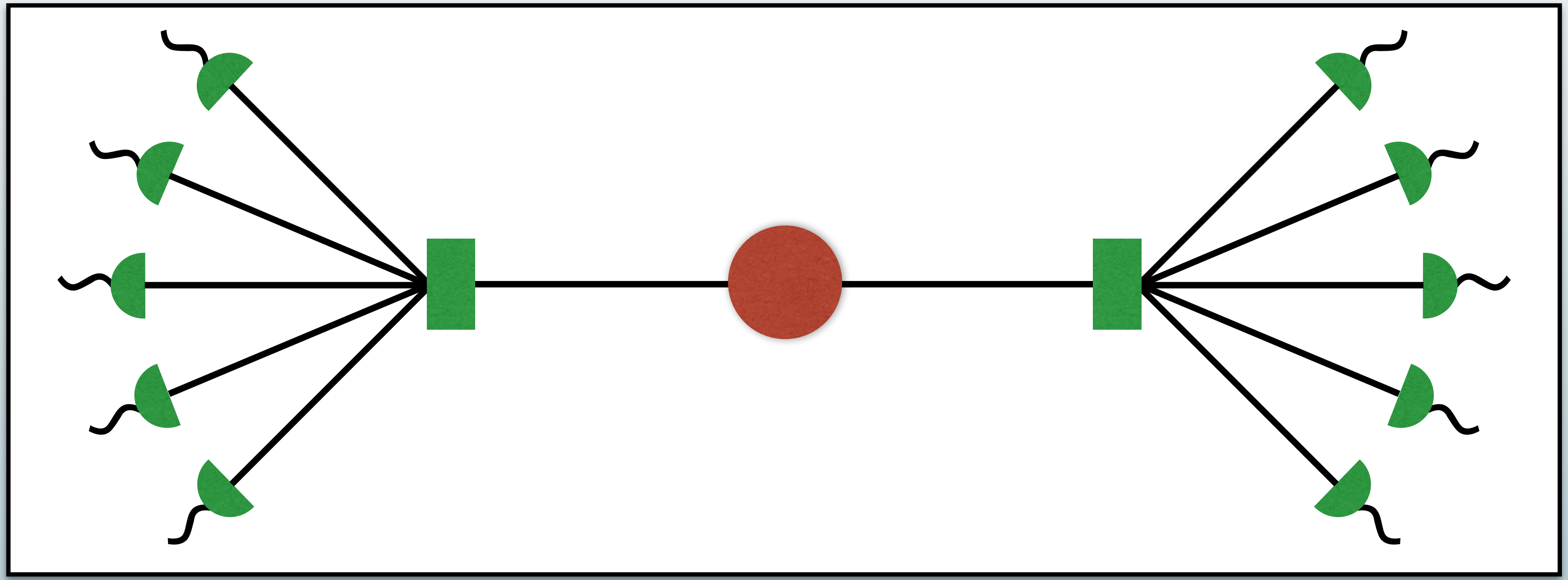


Implementing a d-outcome measurement

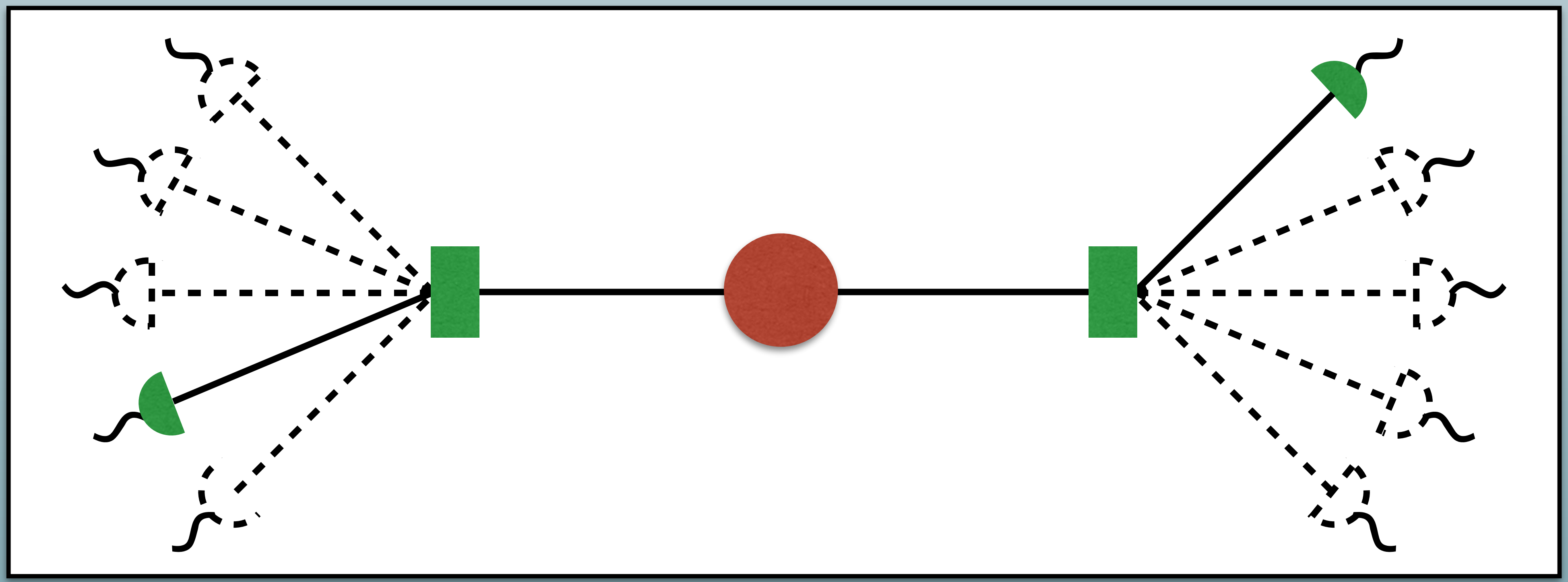


single outcome

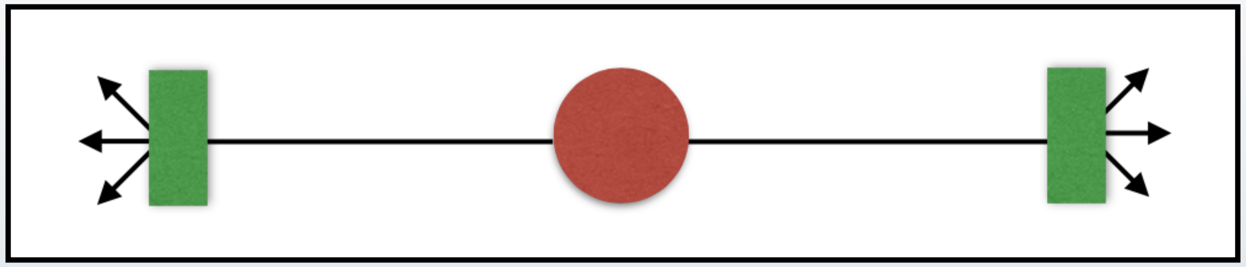
IMPLEMENTING A d-OUTCOME MEASUREMENT



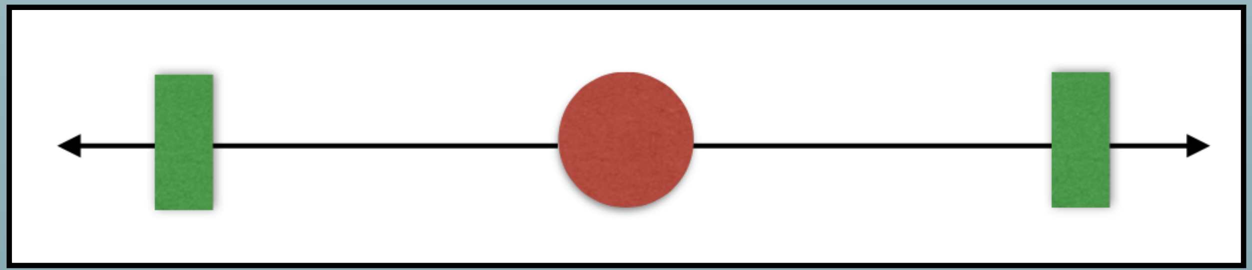
one setting



d^2 settings



Number of outcomes	Tomography	Fidelity	Our Method
d	$(d+1)^2$	$d+1$	2
single	$(d+1)^2d^2$	$(d+1)d^2$	$2d^2$



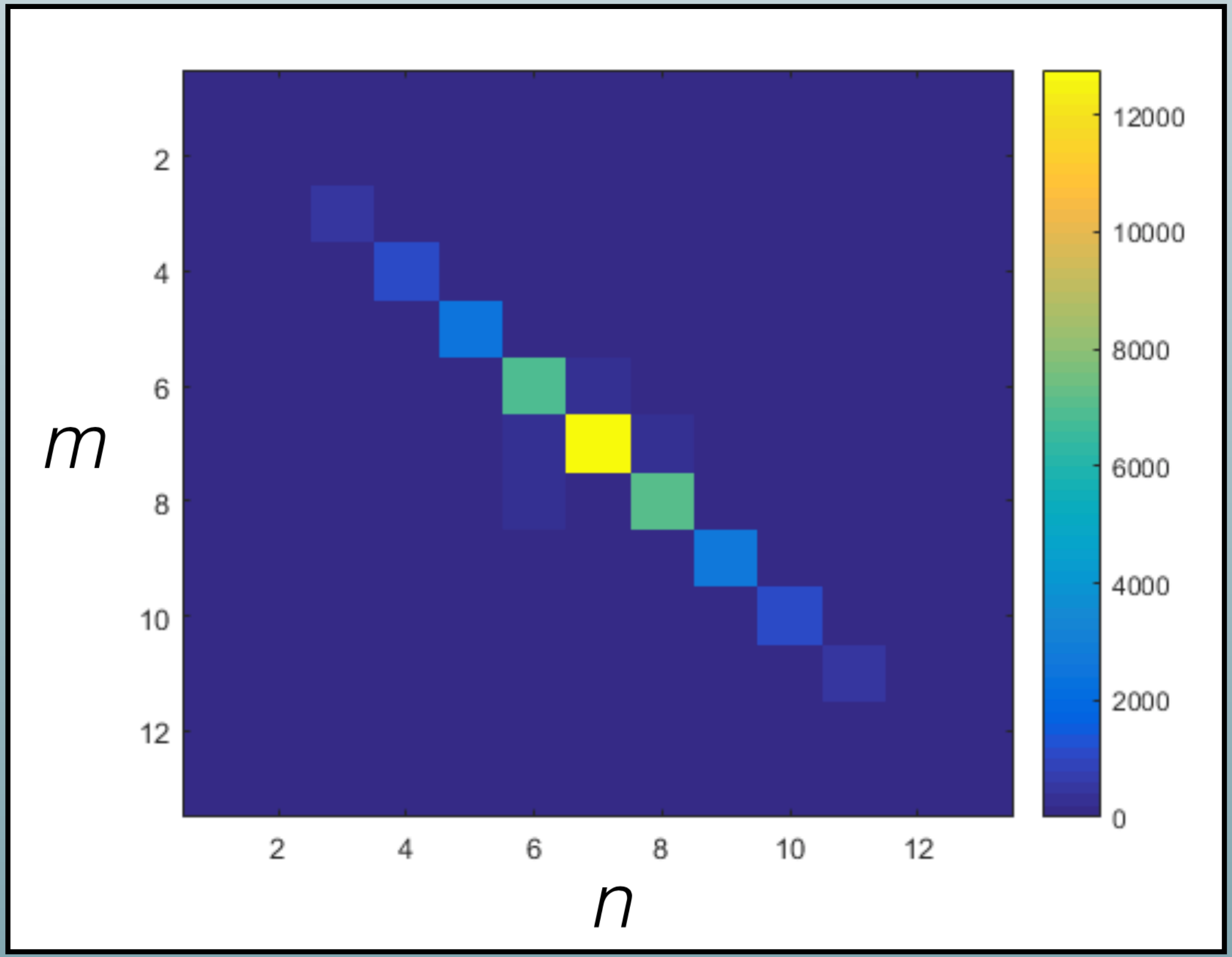
$$\tilde{F}(\rho, \Phi) \leq F(\rho, \Phi) \leq B_k(\Phi)$$

How to choose $|\Phi\rangle$?

Choose *standard basis* $\{|mn\rangle\}$ and measure experimental (unknown) state ρ :

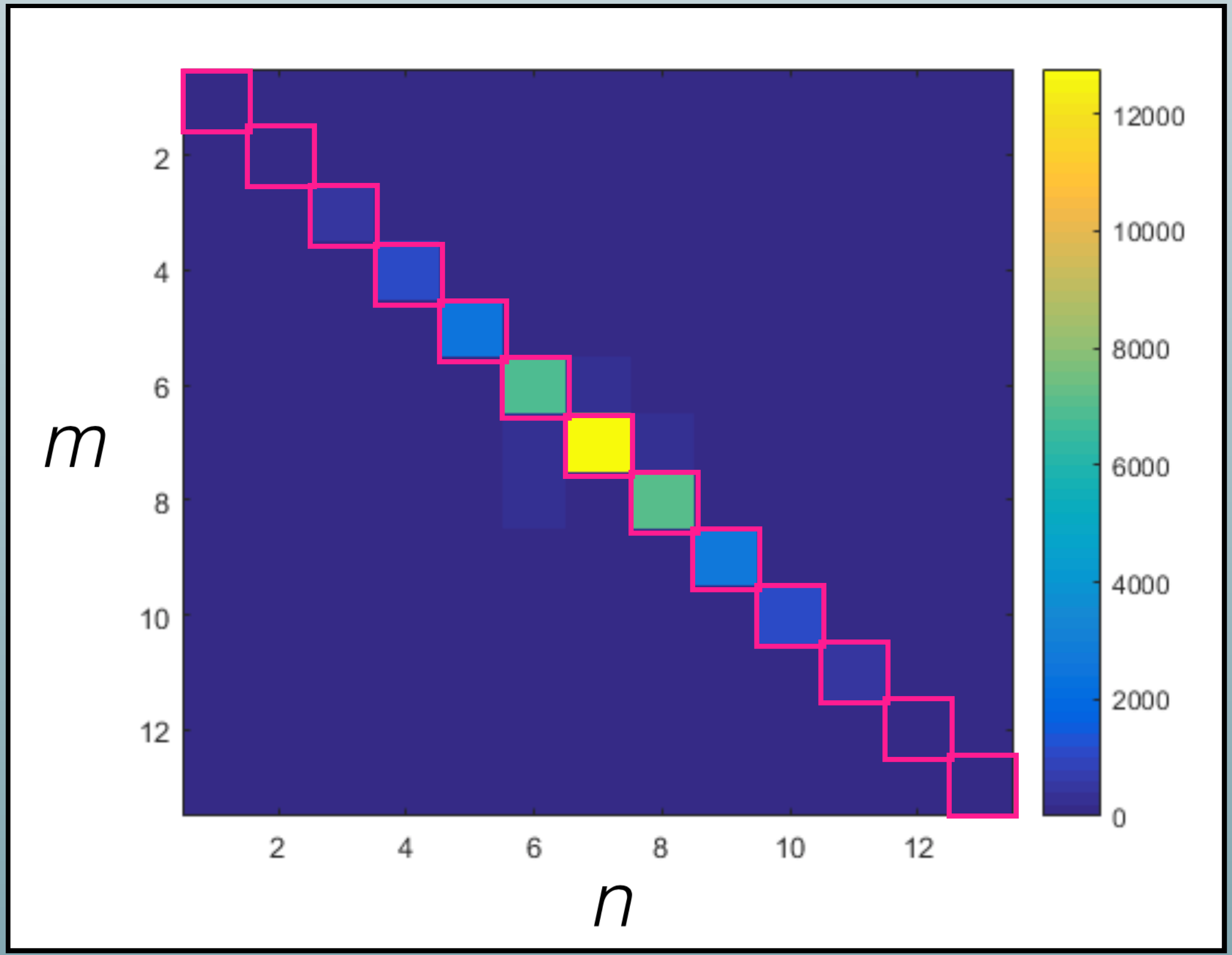
$$\langle mn|\rho|mn\rangle = \frac{N_{mn}}{\sum_{ij} N_{ij}}$$

1.



Calculate $\{\lambda_m\}$:

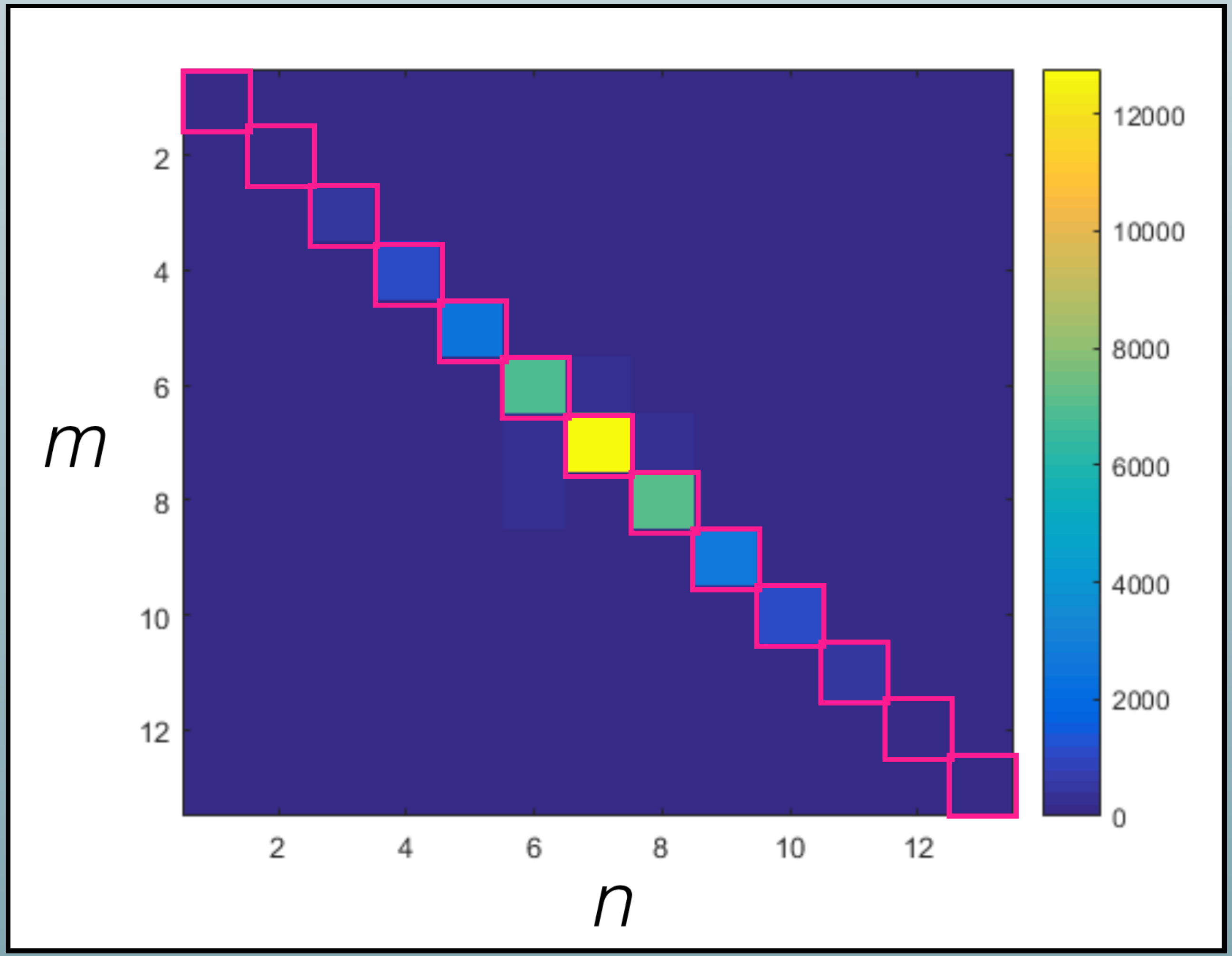
$$\lambda_m = \sqrt{\frac{\langle mm|\rho|mm\rangle}{\sum_n \langle nn|\rho|nn\rangle}}$$



2.

Nominate target state $|\Phi\rangle$:

$$|\Phi\rangle = \sum_{m=0}^{k-1} \lambda_m |mm\rangle$$



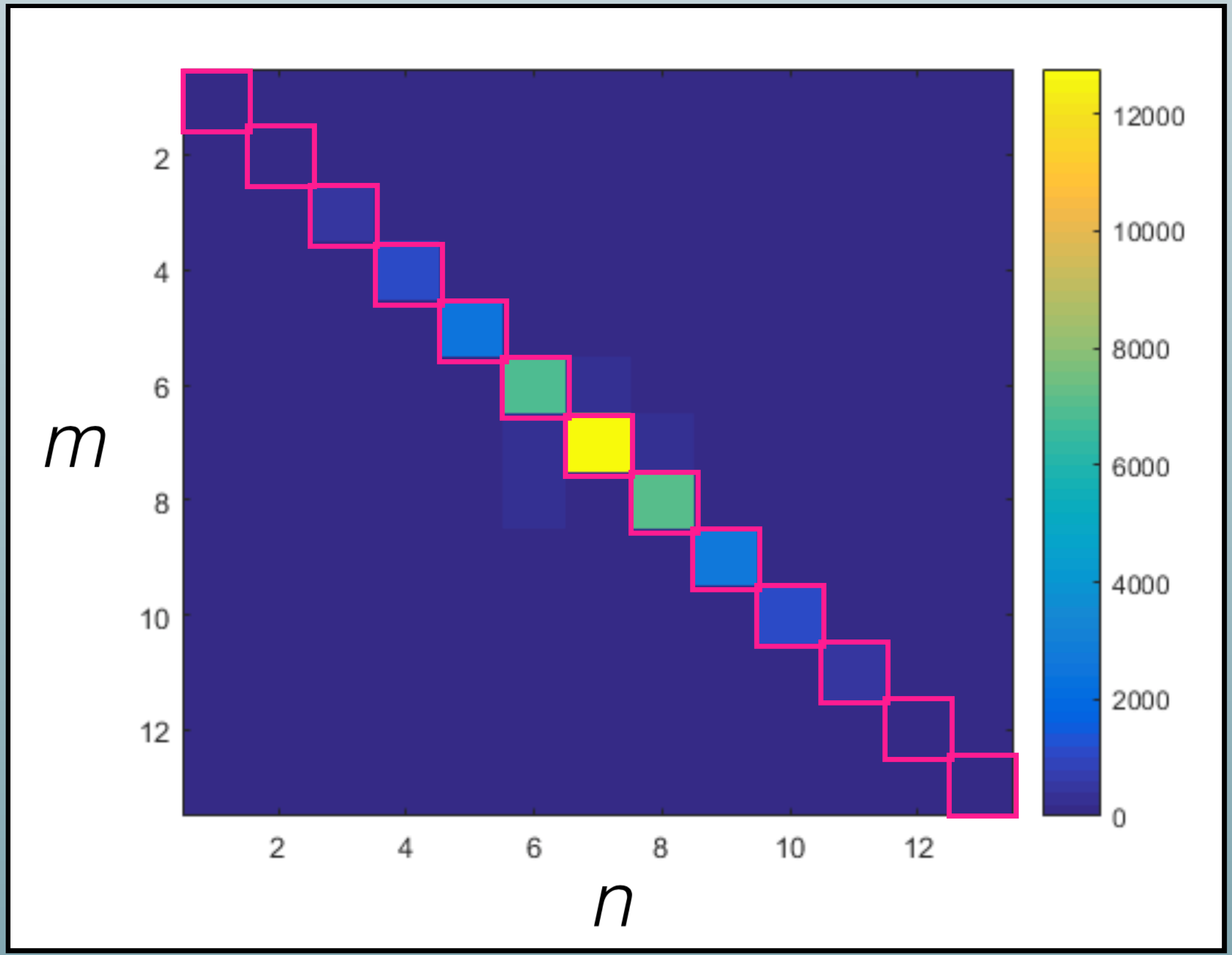
2.

Define *tilted basis* $\{|\tilde{i}\tilde{j}^*\rangle\}$:

$$|\tilde{j}\rangle = \frac{1}{\sqrt{\sum_n \lambda_n}} \sum_{m=0}^{d-1} \omega^{jm} \sqrt{\lambda_m}\,|m\rangle$$

$$|\langle m|\tilde{j}\rangle|^2 = \lambda_m \lambda_j$$

2.

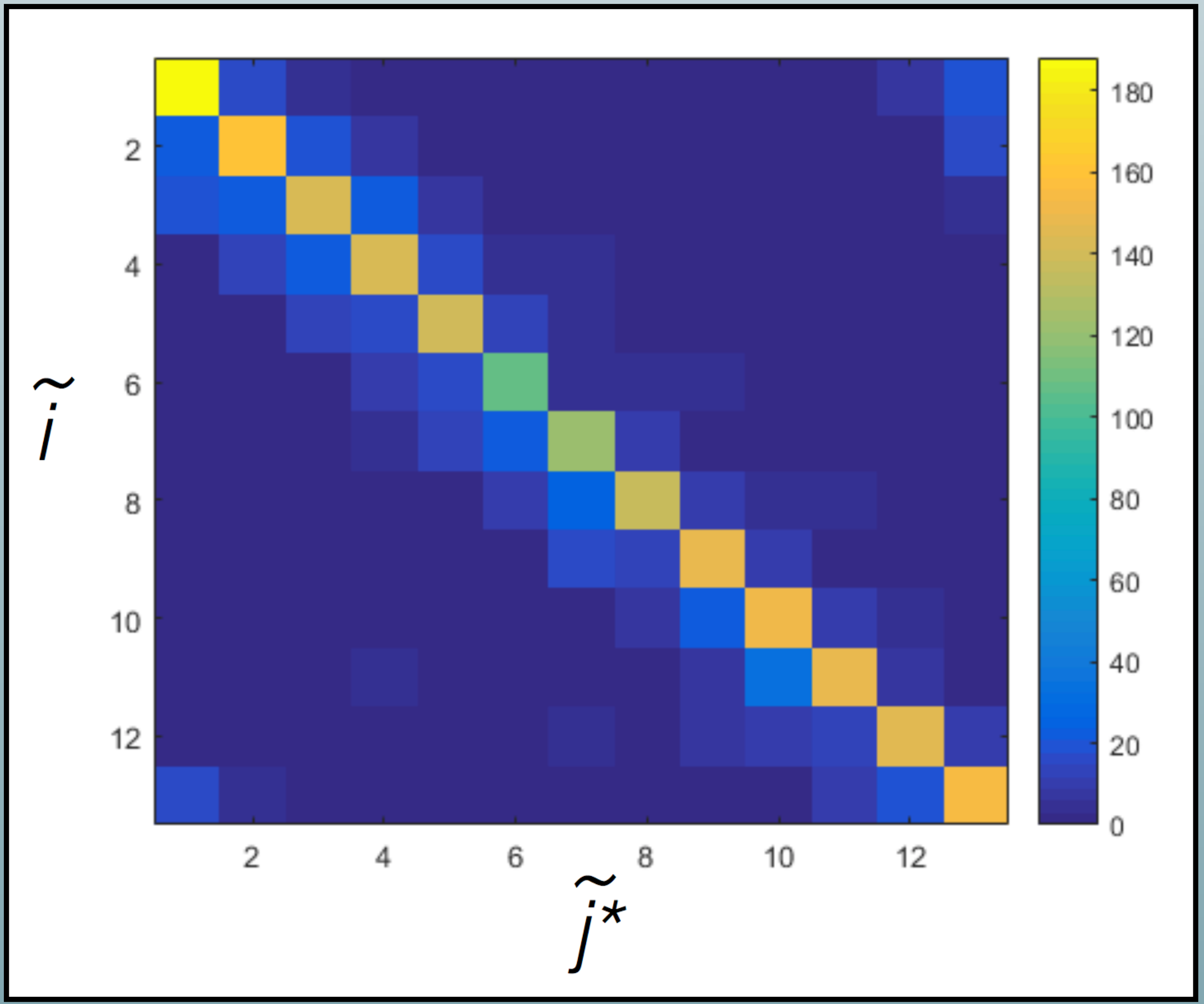


Measure in the tilted basis:

$$\langle \tilde{i}\tilde{j}^* | \rho | \tilde{i}\tilde{j}^* \rangle = \frac{\tilde{N}_{ij}}{\sum_{m,n} \tilde{N}_{mn}} c_\lambda$$

$$c_\lambda = \frac{d^2}{(\sum_k \lambda_k)^2} \sum_{m,n} \lambda_m \lambda_n \langle mn | \rho | mn \rangle$$

3.



Calculate fidelity lower bound and check witness:

$$\tilde{F}(\rho,\Phi) \leq F(\rho,\Phi) \leq B_k(\Phi)$$

$$\tilde{F}(\rho,\Phi) := \frac{(\sum_k \lambda_k)^2}{d} \sum_j \langle \tilde{j}\tilde{j}^* | \rho | \tilde{j}\tilde{j}^* \rangle - \sum_{m,n\neq m} \lambda_m \lambda_n \langle mn|\rho|mn\rangle + \\ - \sum_{\substack{m\neq n, m'\neq m, \\ n\neq n', n'\neq m' \\ m-n-m'+n' \bmod d \neq 0}} \sqrt{\lambda_m \lambda_{m'} \lambda_n \lambda_{n'} \langle mn|\rho|mn\rangle \langle m'n'|\rho|m'n'\rangle}$$

4.

Calculate fidelity lower bound and check witness:

$$\tilde{F}(\rho, \Phi) \leq F(\rho, \Phi) \leq B_k(\Phi)$$

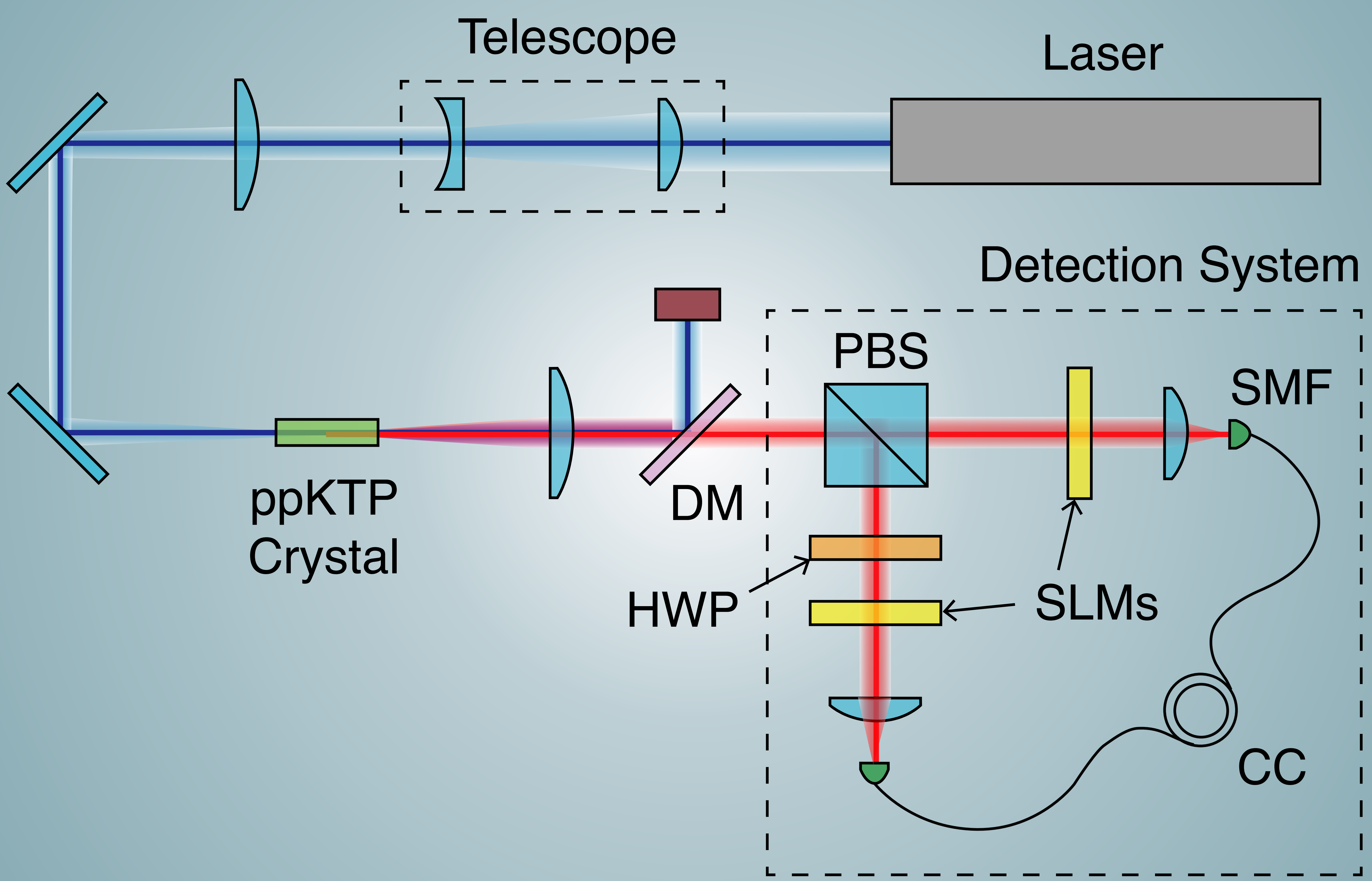
4.

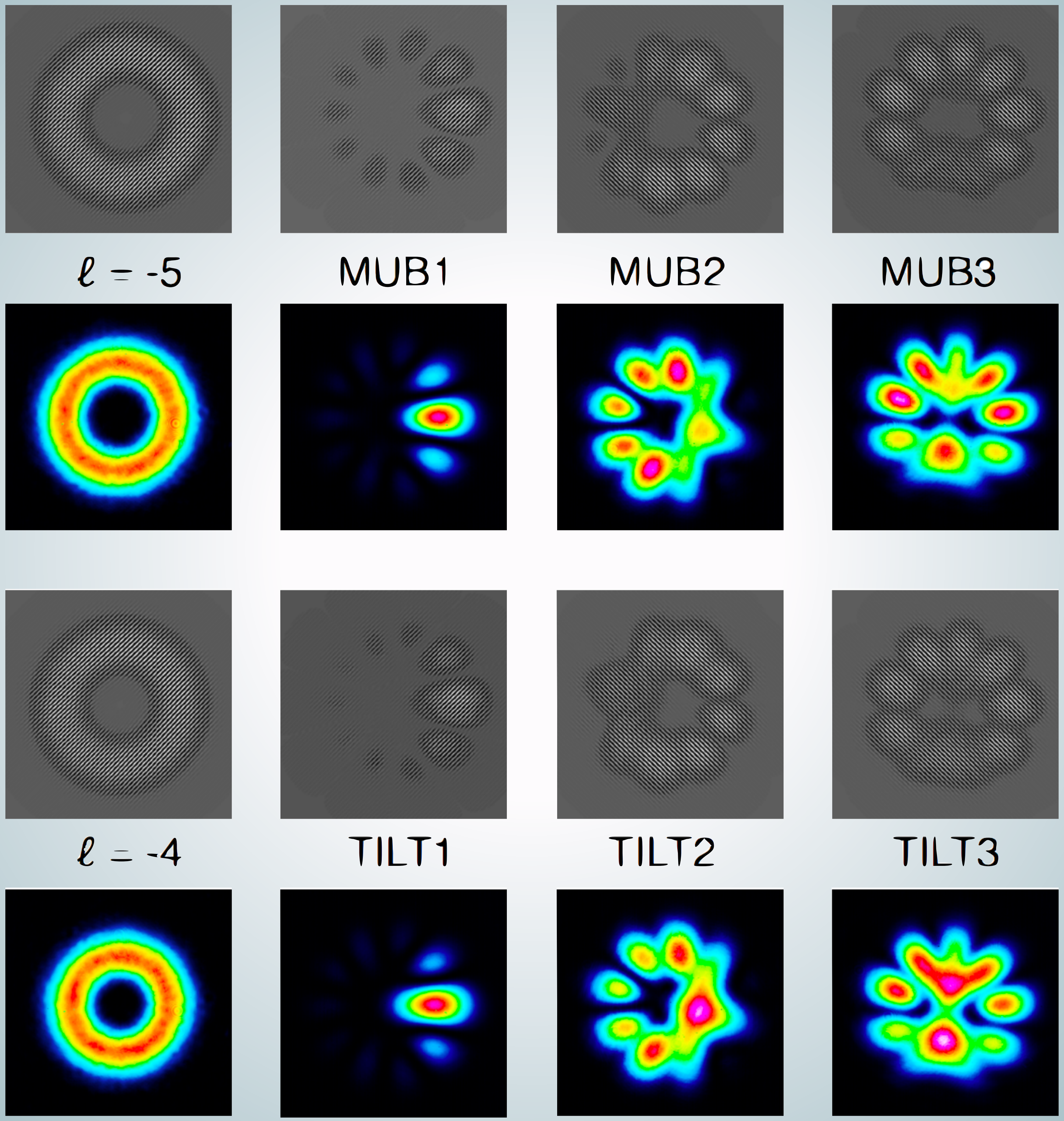
$$\tilde{F}(\rho, \Phi) > B_{k'}(\Phi) \implies k(\rho) = k' + 1$$

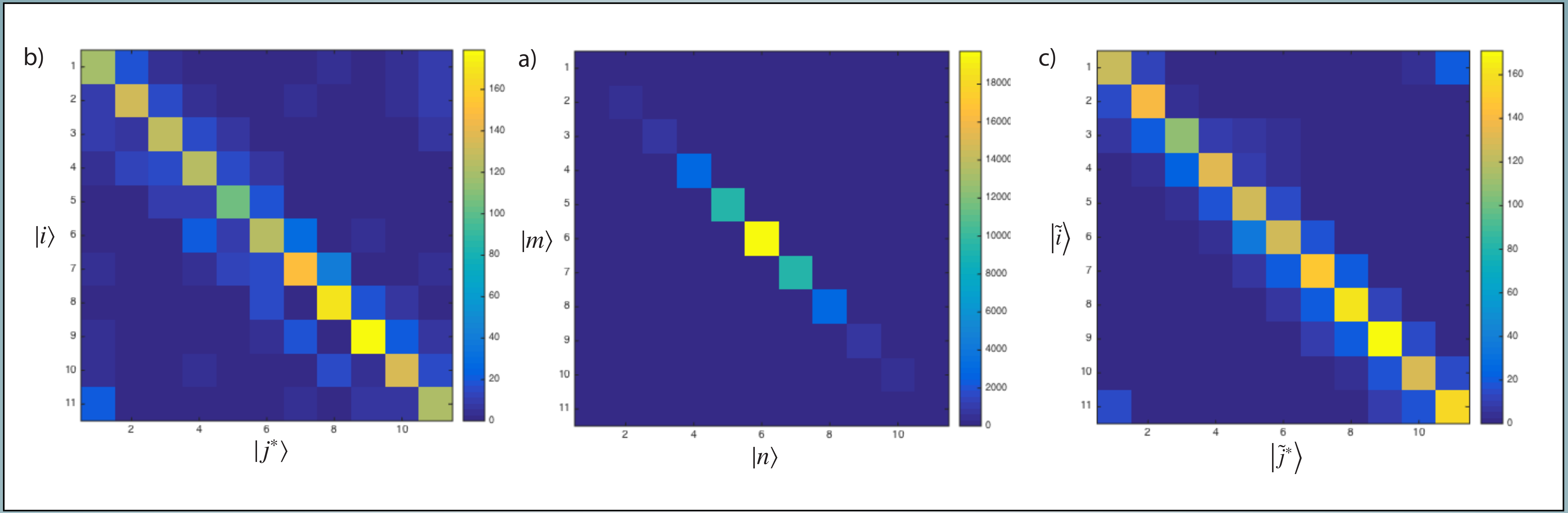
$$\tilde{F}(\rho, \Phi) \leq F(\rho, \Phi) \leq B_k(\Phi)$$

- Exact for pure states and dephased pure states.
- Generalized for M measurements.
- Exact in prime dimensions for $M=d+1$.
- Generalized for multipartite states.
- Provides lower bound for entanglement of formation.

EXPERIMENT


Telescope
Laser
Detection System
PBS
SMF
ppKTP
Crystal
DM
HWP
SLMs
CC


ℓ = -5
MUB1
MUB2
MUB3
ℓ = -4
TILT1
TILT2
TILT3

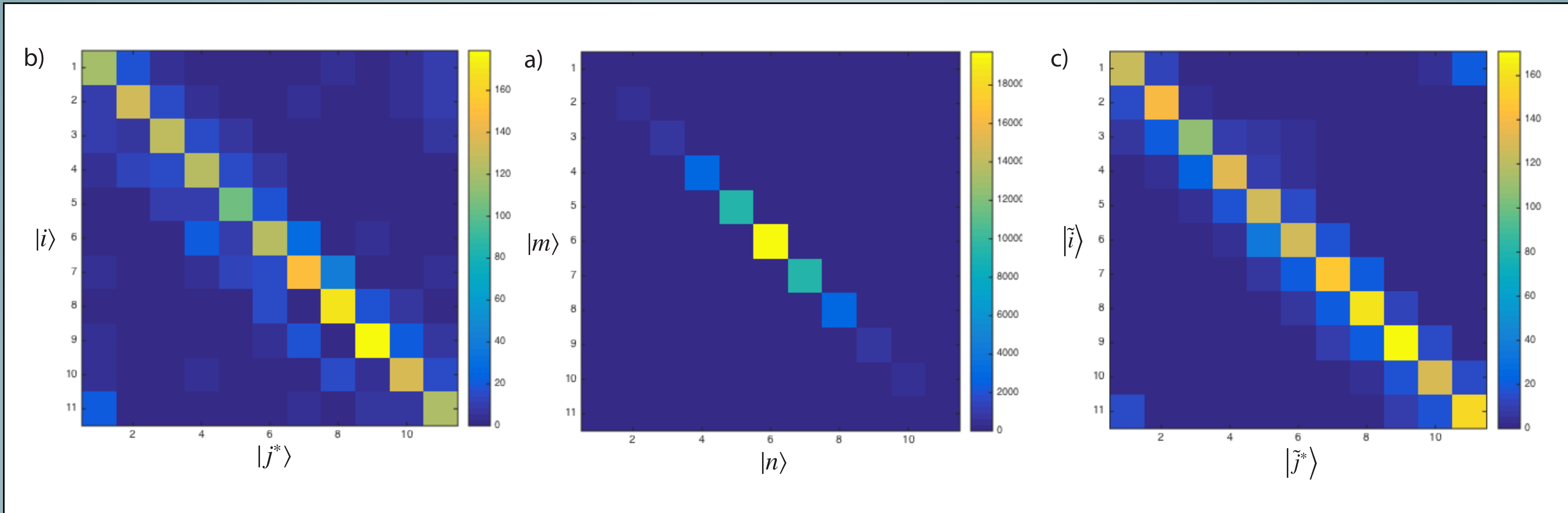


MUB

COMP. BASIS

TILTED BASIS

$$\tilde{F}(\rho, |\Phi^+\rangle) \qquad \tilde{F}(\rho, |\Phi\rangle)$$



d	d_{ent}	$\tilde{F}(\rho, \Phi^+)$	$\tilde{F}(\rho, \Phi)$
3	3	91.5±0.4%	92.5±0.4%
5	5	89.9±0.4%	90.0±0.5%
7	6	84.2±0.5%	86.9±0.6%
11	9	74.8±0.4%	76.2 ±0.6%

Summary of Results

Theory

- Fidelity and Schmidt number certification with two measurements without assumptions on the state.

Experiment

- Highest Schmidt number ($k=9$) ever certified without assumptions on the state.

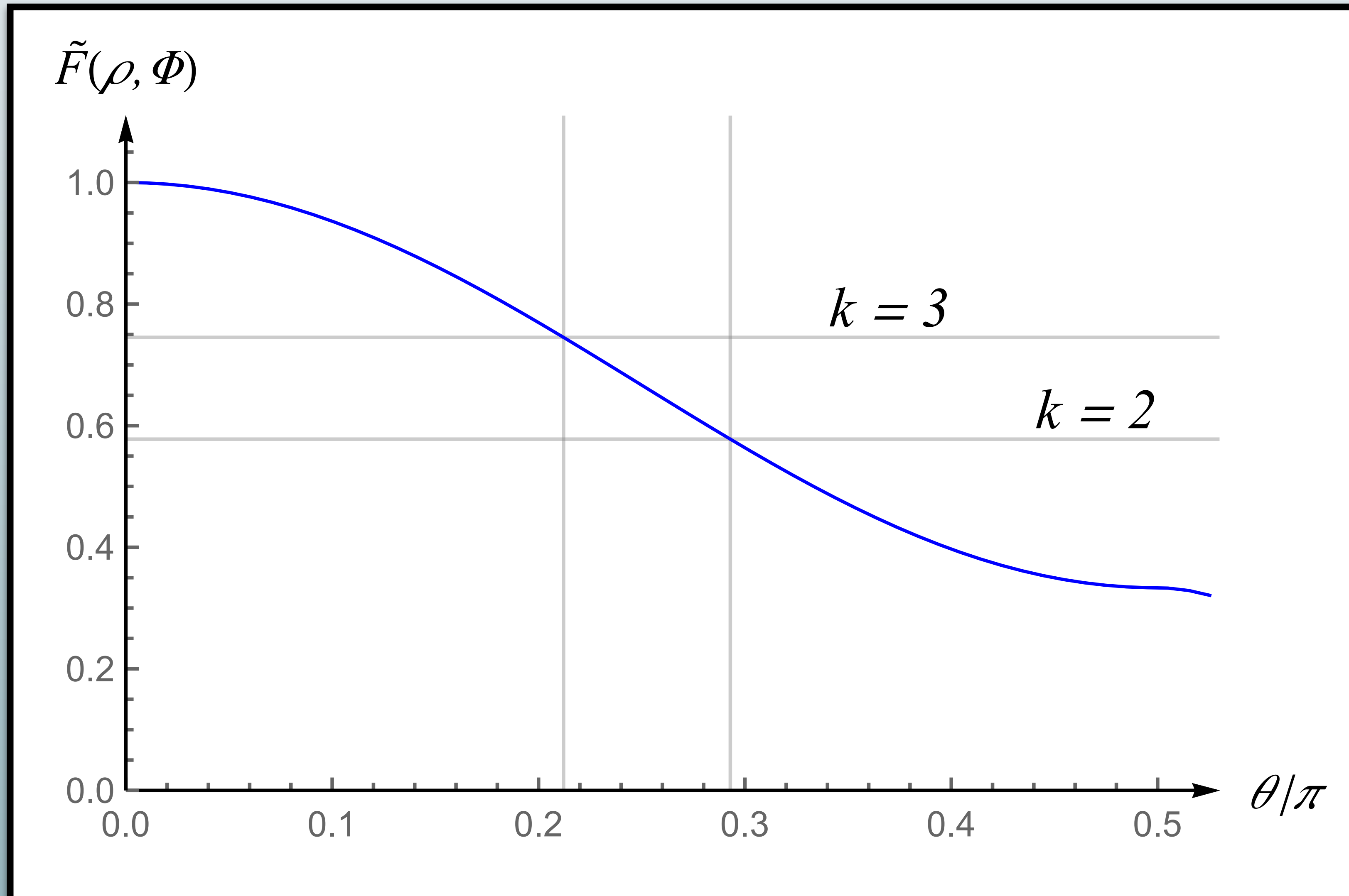
THANK YOU!

EXTRA SLIDES

$$|\tilde{j}\rangle = \frac{1}{\sqrt{\sum_n \lambda_n}} \sum_{m=0}^{d-1} \omega^{jm} \sqrt{\lambda_m}\, |m\rangle$$

$$\sum_{i,j} \langle \tilde{i}\tilde{j}^* | \rho | \tilde{i}\tilde{j}^* \rangle = \underbrace{\frac{d^2}{(\sum_k \lambda_k)^2} \sum_{m,n} \lambda_m \lambda_n \langle mn | \rho | mn \rangle}_{c_\lambda :=}$$

Deviation of the Schmidt Basis



NOISE RESISTANCE