SEMI-DEVICE-INDEPENDENT CERTIFICATION OF

INDEFINITE CAUSAL ORDER

JESSICA BAVARESCO, MATEUS ARAÚJO, ČASLAV BRUKNER, MARCO TÚLIO QUINTINO

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IQOQI Vienna

ÖAW ÖSTERREICHISCHE AKADEMIE DER WISSENSCHAFTEN

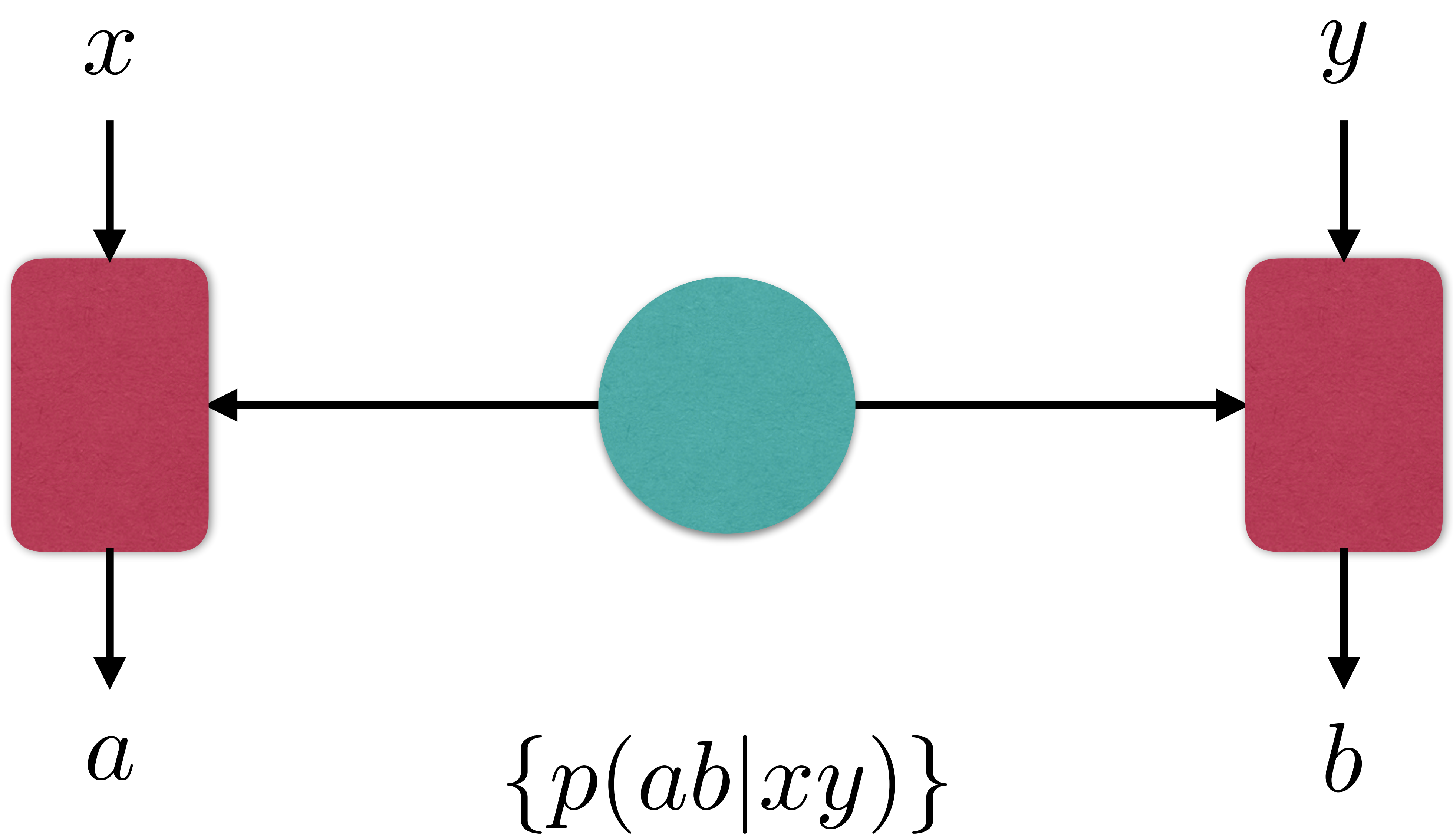
MOTIVATION

Given a set of probability distributions (behaviour)

$$\{p(ab|xy)\}$$

what conclusions can be taken from it by making different assumptions about how it was obtained?

MOTIVATION



MOTIVATION

$$\{p(ab|xy)\}$$

Assumed to be known:
(trusted)

Unknown/variables:
(untrusted)

MOTIVATION

$$\{p(ab|xy)\}$$

Assumed to be known:
(trusted)

$$\{A_{a|x}\}$$

$$\{B_{b|y}\}$$

Unknown/variables:
(untrusted)

$$\rho$$

MOTIVATION

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Assumed to be known:
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$$\{A_{a|x}\}$$

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Unknown/variables:
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$$\rho$$

$$p(ab|xy) \overset{?}{=} \mathrm{Tr}\left[\left(A_{a|x} \otimes B_{b|y}\right) \rho_A \otimes \rho_B\right]$$

MOTIVATION

$$\{p(ab|xy)\}$$

Assumed to be known:
(trusted)

$$\{A_{a|x}\}$$

$$\{B_{b|y}\}$$

Unknown/variables:
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$$\rho$$

$$p(ab|xy) \stackrel{?}{=} \mathrm{Tr}\left[\left(A_{a|x} \otimes B_{b|y}\right) \rho_{\mathrm{SEP}}\right]$$

MOTIVATION

$$\{p(ab|xy)\}$$

Assumed to be known:
(trusted)

$$\{A_{a|x}\}$$

$$\{B_{b|y}\}$$

Unknown/variables:
(untrusted)

$$\rho$$

$$p(ab|xy) \stackrel{?}{=} \mathrm{Tr}\left[\left(A_{a|x} \otimes B_{b|y}\right) \rho_{AB}\right]$$

$$p(ab|xy) = \mathrm{Tr}\left[\left(A_{a|x} \otimes B_{b|y}\right)\, \rho_{AB}\right]$$

and

$$p(ab|xy) \neq \mathrm{Tr}\left[\left(A_{a|x} \otimes B_{b|y}\right)\, \rho_{\mathrm{SEP}}\right]$$

$$p(ab|xy) = \mathrm{Tr}\left[\left(A_{a|x} \otimes B_{b|y}\right)\ \rho_{AB}\right]$$

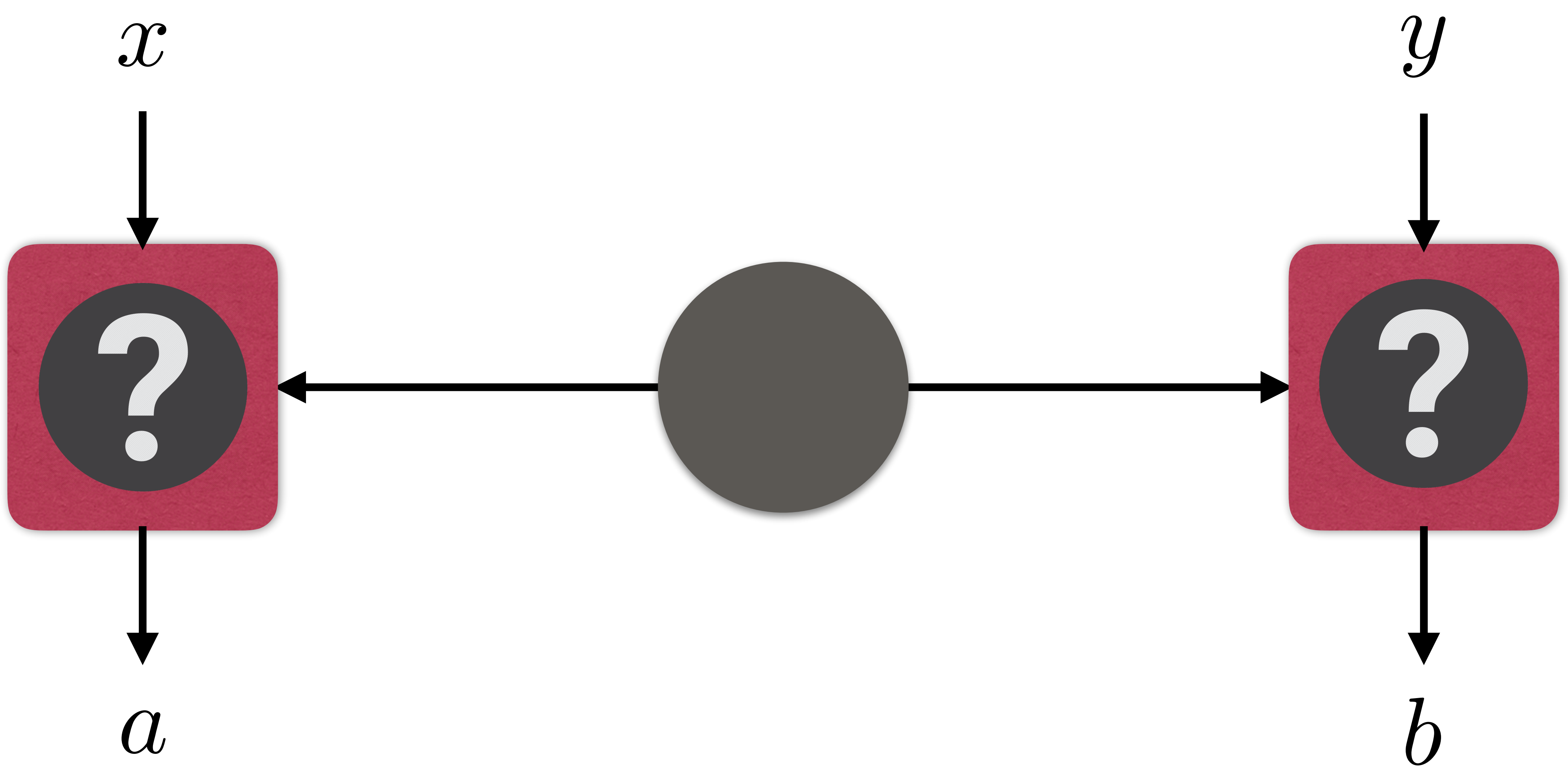
and

$$p(ab|xy) \neq \mathrm{Tr}\left[\left(A_{a|x} \otimes B_{b|y}\right)\ \rho_{\mathrm{SEP}}\right]$$

$$\Downarrow$$

ρ_{AB} is entangled

DEVICE DEPENDENCE



DEVICE INDEPENDENT

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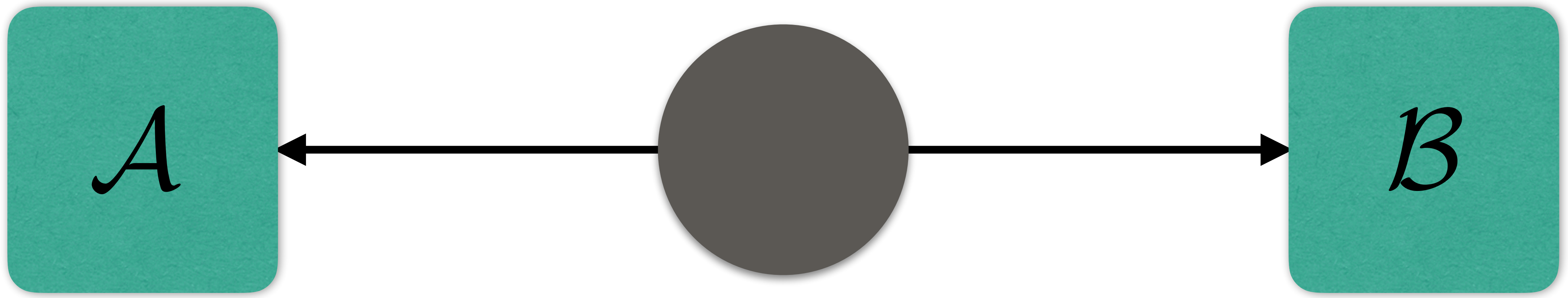

DEVICE DEPENDENT

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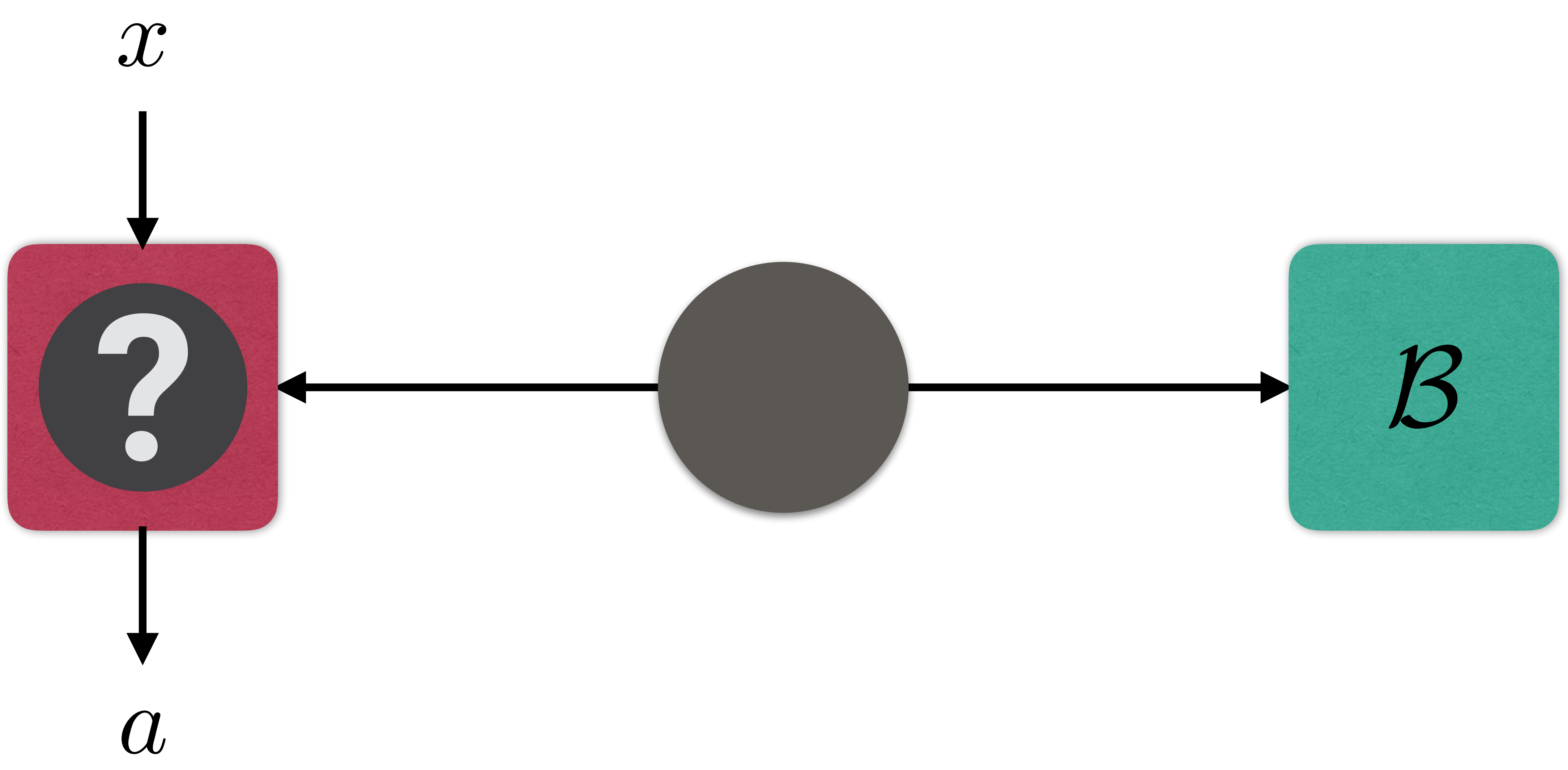
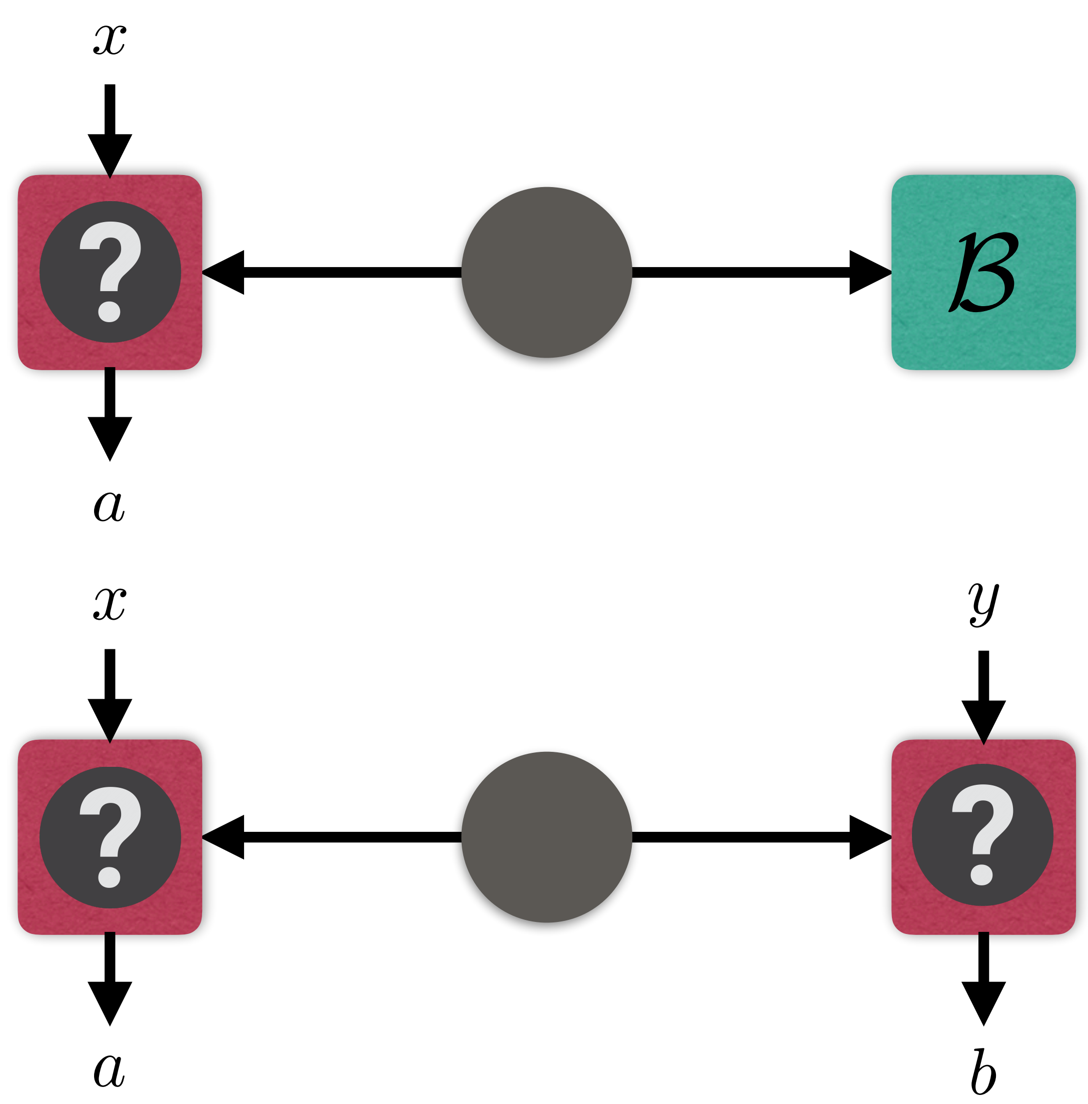


SEMI-DEVICE INDEPENDENT

EXAMPLE: ENTANGLEMENT



DEVICE DEPENDENT	ENTANGLEMENT WITNESS
SEMI-DEVICE INDEPENDENT	EPR STEERING
DEVICE INDEPENDENT	BELL NONLOCALITY

INDEFINITE CAUSAL ORDER

PROCESS MATRIX FORMALISM

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- Most general operation in quantum mechanics: a set of instruments

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- Most general operation in quantum mechanics: a set of instruments

$$\{I_{a|x}\},\ I_{a|x} \in \mathcal{L}(\mathcal{H}^I \otimes \mathcal{H}^O)$$

$$I_{a|x} \geq 0, \quad \forall\ a, x$$

$$\mathrm{Tr}_O \sum_a I_{a|x} = \mathbb{1}^I, \quad \forall\ x,$$

$x \in \{1, ..., I\}$ *labels the instruments*

$a \in \{1, ..., O\}$ *labels their outcomes*

PROCESS MATRIX FORMALISM

- Extracting sets of probability distributions from instruments:

PROCESS MATRIX FORMALISM

- Extracting sets of probability distributions from instruments:

The most general bilinear function $f : (A_{a|x}, B_{b|y}) \to \mathbb{R}$ *that extracts valid sets of probability distributions from sets of quantum instruments* $\{A_{a|x}\}, A_{a|x} \in H^{A_I} \otimes H^{A_O}$ *and* $\{B_{b|y}\}, B_{b|y} \in H^{B_I} \otimes H^{B_O}$ *is*

$$p(ab|xy) = \mathrm{Tr}[(A_{a|x} \otimes B_{b|y})\, W],$$

where $W \in H^{A_I} \otimes H^{A_O} \otimes H^{B_I} \otimes H^{B_O}$ *is a* ***process matrix.***

O. Oreshkov, F. Costa, C. Brukner, Nat. Comm. 3, 1092 (2012)

PROCESS MATRIX FORMALISM

$$p(ab|xy) = \mathrm{Tr}\left[(A_{a|x}^{A_I A_O A'} \otimes B_{b|y}^{B_I B_O B'}) \, W^{A_I A_O B_I B_O} \otimes \rho^{A' B'} \right]$$

O. Oreshkov, F. Costa, C. Brukner, Nat. Comm. 3, 1092 (2012)

PROCESS MATRIX FORMALISM

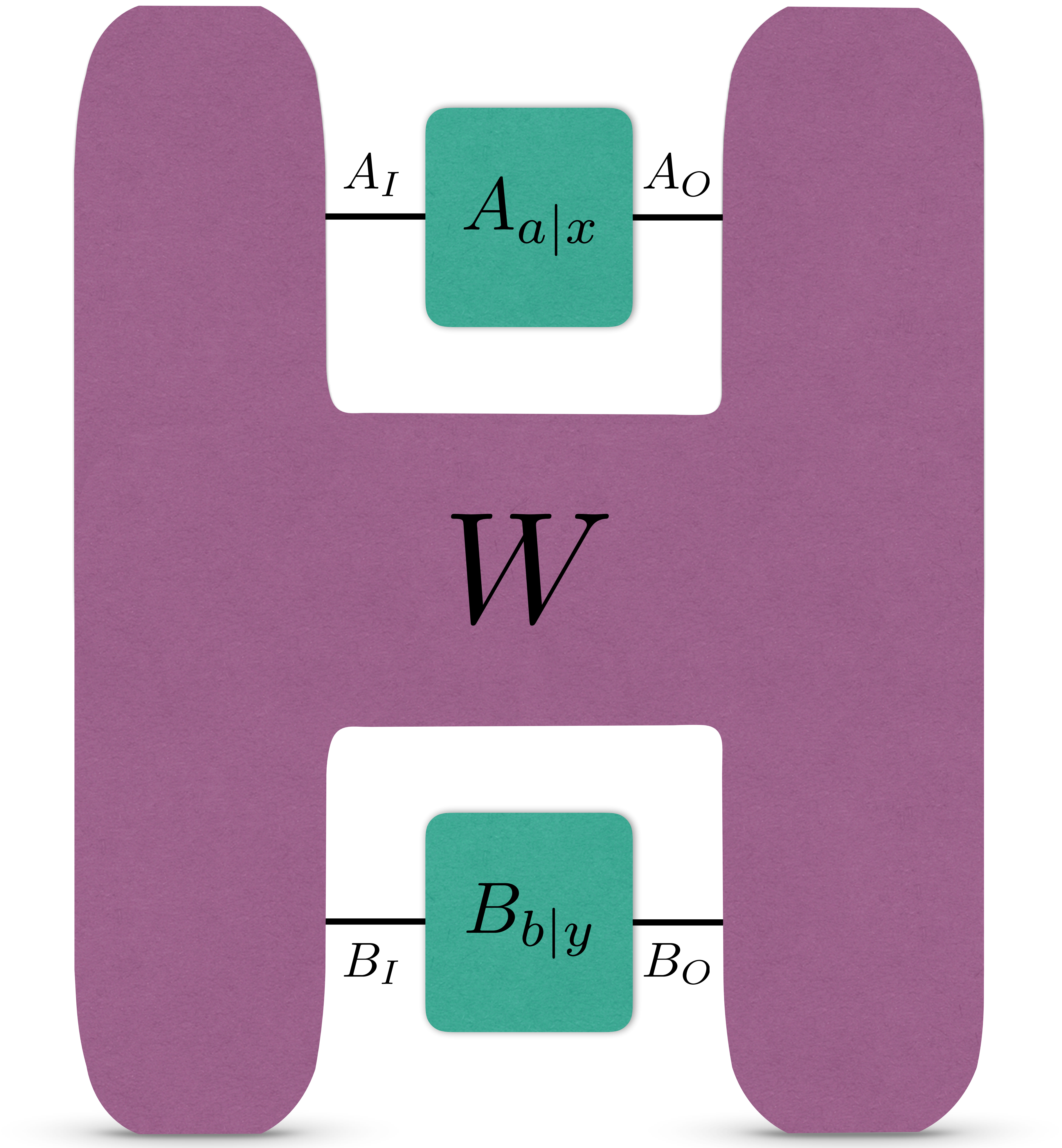
$$W \in H^{A_I} \otimes H^{A_O} \otimes H^{B_I} \otimes H^{B_O}$$

$$\begin{aligned} W &\geq 0 \\ \mathrm{Tr}\, W &= d_{A_O} d_{B_O} \\ {}_{A_I A_O} W &= {}_{A_I A_O B_O} W \\ {}_{B_I B_O} W &= {}_{A_O B_I B_O} W \\ W &= {}_{A_O} W + {}_{B_O} W - {}_{A_O B_O} W, \end{aligned}$$

where ${}_X W := \mathrm{Tr}_X W \otimes \frac{\mathbb{1}^X}{d_X}$ is the trace-and-replace operation.

M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, C. Brukner, New J. Phys. 17, 102001 (2015)

PROCESS MATRIX FORMALISM



CAUSALITY PROPERTIES

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- At the level of probability distributions:

CAUSALITY PROPERTIES

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A behaviour $\{p^{A \prec B}(ab|xy)\}$ *is* ***causally ordered*** *from Alice to Bob if it satisfies*

$$\sum_b p^{A \prec B}(ab|xy) = \sum_b p^{A \prec B}(ab|xy'), \qquad \forall\, a, x, y, y'$$

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- At the level of probability distributions:

A behaviour $\{p^{A\prec B}(ab|xy)\}$ *is* ***causally ordered*** *from Alice to Bob if it satisfies*

$$\sum_b p^{A\prec B}(ab|xy) = \sum_b p^{A\prec B}(ab|xy'), \qquad \forall\, a, x, y, y'$$

A behaviour $\{p^{\text{causal}}(ab|xy)\}$ *is* ***causal*** *if it can be expressed as a conv. comb. of ordered behaviours, i.e.,*

$$p^{\text{causal}}(ab|xy) := q\, p^{A\prec B}(ab|xy) + (1-q)p^{B\prec A}(ab|xy), \qquad \forall\, a, b, x, y$$

CAUSALITY PROPERTIES

- At the level of process matrices:

CAUSALITY PROPERTIES

- At the level of process matrices:

A process matrix $W^{A\prec B}$ *that is* ***causally ordered*** *from Alice to Bob is the most general operator that takes pairs of instruments to behaviours that are causally ordered from Alice to Bob, according to*

$$p^{A\prec B}(ab|xy) = \mathrm{Tr}\left[\left(A_{a|x} \otimes B_{b|y}\right) W^{A\prec B}\right].$$

O. Oreshkov, F. Costa, C. Brukner, Nat. Comm. 3, 1092 (2012)

CAUSALITY PROPERTIES

- At the level of process matrices:

$$W^{A \prec B}$$

CAUSALITY PROPERTIES

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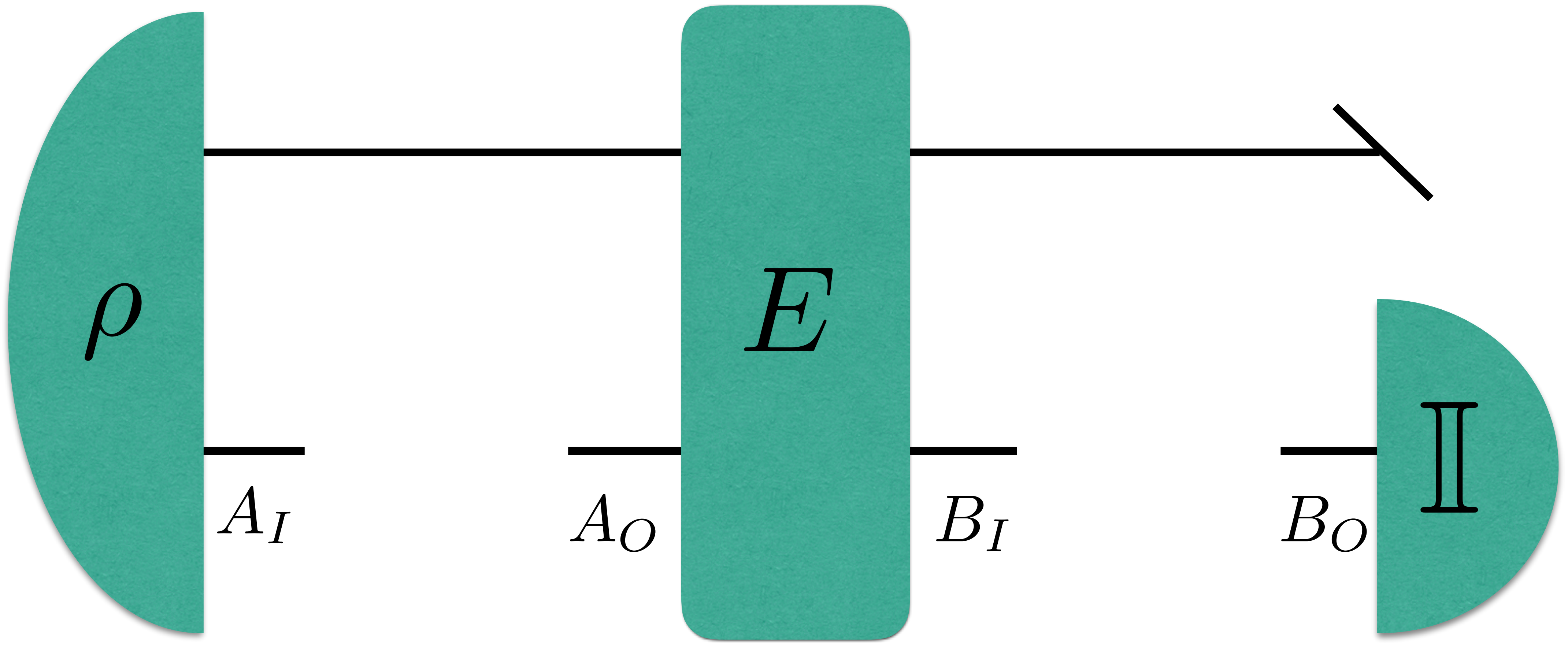
$$W^{A \prec B}$$



G. Chiribella, G. M. D'Ariano, P. Perinotti, EPL 83, 3 (2008)

CAUSALITY PROPERTIES

- At the level of process matrices:

A process matrix W^{sep} *is **causally separable** if it can be expressed as a convex combination of ordered process matrices, i.e.,*

$$W^{\mathrm{sep}} =: qW^{A \prec B} + (1-q)W^{B \prec A},$$

where $0 \leq q \leq 1$ *is a real number.*

CAUSALITY PROPERTIES

$$W \neq q\,W^{A\prec B} + (1-q)W^{B\prec A}$$

CAUSALITY PROPERTIES

Known advantages of indefinite causal order:

- Channel discrimination: G. Chiribella, *PRA* **86**, 040301 (2012)
- Quantum computation: M. Araújo, *et al.*, *PRL* **113**, 250402 (2014)
- Communication complexity: P. A. Guérin, *et al.*, *PRL* **117**, 100502 (2016)
- Enhanced channel capacity: D. Ebler, *et al.*, *PRL* **120**, 120502 (2018)
- Unitary inversion: M. T. Quintino, *et al.*, *PRL* **123**, 210502 (2019)

THEORY GAP

DEVICE DEPENDENT	SEMI-DEVICE INDEPENDENT	DEVICE INDEPENDENT

THEORY GAP

DEVICE DEPENDENT	SEMI-DEVICE INDEPENDENT	DEVICE INDEPENDENT
THEORETICAL FRAMEWORK (CAUSAL WITNESSES)[1]		

THEORY GAP

DEVICE DEPENDENT	SEMI-DEVICE INDEPENDENT	DEVICE INDEPENDENT
THEORETICAL FRAMEWORK (CAUSAL WITNESSES)[1]		THEORETICAL FRAMEWORK (CAUSAL INEQUALITIES)[3]

THEORY GAP

DEVICE DEPENDENT	SEMI-DEVICE INDEPENDENT	DEVICE INDEPENDENT
THEORETICAL FRAMEWORK (CAUSAL WITNESSES)[1]		THEORETICAL FRAMEWORK (CAUSAL INEQUALITIES)[3]
SOME EXPERIMENTAL PROPOSALS AND IMPLEMENTATIONS[2]		

THEORY GAP

DEVICE DEPENDENT	SEMI-DEVICE INDEPENDENT	DEVICE INDEPENDENT
THEORETICAL FRAMEWORK (CAUSAL WITNESSES)[1]		THEORETICAL FRAMEWORK (CAUSAL INEQUALITIES)[3]
SOME EXPERIMENTAL PROPOSALS AND IMPLEMENTATIONS[2]		NO KNOWN EXPERIMENTAL IMPLEMENTATION

[1] M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, C. Brukner, New J. Phys. 17, 102001 (2015)

[2] L. M. Procopio, et al., Nat. Comm. 6, 7913 (2015). G. Rubino, et al., Science Advances 3, 3 (2017). K. Goswami et al., PRL 121, 090503 (2018).

[3] C. Branciard, M. Araújo, A. Feix, F. Costa, Č. Brukner, New, J, Phys. 18, 013008 (2016)

GOALS

- Efficiently check whether a given behaviour certifies that the process matrix that gave rise to it is causally non-separable in a DD, SDI, and DI way.

- Characterise which sets of causally non-separable process matrices can be certified in each scenario.

Device-dependent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$	W
$\{\overline{A}_{a\|x}\}$, $\{\overline{B}_{b\|y}\}$	

Device-dependent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$	W
$\{\overline{A}_{a\|x}\}$, $\{\overline{B}_{b\|y}\}$	

$$p^Q(ab|\overline{A}_{a|x}, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(\overline{A}_{a|x} \otimes \overline{B}_{b|y}) W^{\mathrm{sep}}\right]$$

$$\forall\, W^{\mathrm{sep}}$$

Device-dependent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{A}_{a\|x}\}$, $\{\overline{B}_{b\|y}\}$	W

Semi-device-independent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{B}_{b\|y}\}$	d_{A_I}, d_{A_O} $\{A_{a\|x}\}$ W

$$p^Q(ab|\overline{A}_{a|x}, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(\overline{A}_{a|x} \otimes \overline{B}_{b|y}) W^{\mathrm{sep}}\right]$$

$$\forall\, W^{\mathrm{sep}}$$

Device-dependent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{A}_{a\|x}\}$, $\{\overline{B}_{b\|y}\}$	W

Semi-device-independent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{B}_{b\|y}\}$	d_{A_I}, d_{A_O} $\{A_{a\|x}\}$ W

$$p^Q(ab|\overline{A}_{a|x}, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(\overline{A}_{a|x} \otimes \overline{B}_{b|y}) W^{\mathrm{sep}}\right]$$

$$\forall\, W^{\mathrm{sep}}$$

$$p^Q(ab|x, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y}) W^{\mathrm{sep}}\right]$$

$$\forall\, \{A_{a|x}\}, W^{\mathrm{sep}}$$

Device-dependent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{A}_{a\|x}\}$, $\{\overline{B}_{b\|y}\}$	W

Semi-device-independent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{B}_{b\|y}\}$	d_{A_I}, d_{A_O} $\{A_{a\|x}\}$ W

Device-independent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$	$d_{A_I}, d_{A_O}, d_{B_I}, d_{B_O}$ $\{A_{a\|x}\}$, $\{B_{b\|y}\}$ W

$$p^Q(ab|\overline{A}_{a|x}, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(\overline{A}_{a|x} \otimes \overline{B}_{b|y})W^{\mathrm{sep}}\right]$$
$$\forall\, W^{\mathrm{sep}}$$

$$p^Q(ab|x, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y})W^{\mathrm{sep}}\right]$$
$$\forall\, \{A_{a|x}\}, W^{\mathrm{sep}}$$

Device-dependent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{A}_{a\|x}\}$, $\{\overline{B}_{b\|y}\}$	W

$$p^Q(ab|\overline{A}_{a|x}, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(\overline{A}_{a|x} \otimes \overline{B}_{b|y})W^{\mathrm{sep}}\right]$$
$$\forall\, W^{\mathrm{sep}}$$

Semi-device-independent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$ $\{\overline{B}_{b\|y}\}$	d_{A_I}, d_{A_O} $\{A_{a\|x}\}$ W

$$p^Q(ab|x, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y})W^{\mathrm{sep}}\right]$$
$$\forall\, \{A_{a|x}\}, W^{\mathrm{sep}}$$

Device-independent

Given quantities	Variables
$\{p^Q(ab\|xy)\}$	$d_{A_I}, d_{A_O}, d_{B_I}, d_{B_O}$ $\{A_{a\|x}\}$, $\{B_{b\|y}\}$ W

$$p^Q(ab|xy) \neq \mathrm{Tr}\left[(A_{a|x} \otimes B_{b|y})W^{\mathrm{sep}}\right]$$
$$\forall\, \{A_{a|x}\}, \{B_{b|y}\}, W^{\mathrm{sep}}$$

DEVICE DEPENDENT

$$\{p^Q(ab|xy)\}, \{\overline{A}_{a|x}\}, \{\overline{B}_{b|y}\}$$

$$p^Q(ab|\overline{A}_{a|x}, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(\overline{A}_{a|x} \otimes \overline{B}_{b|y}) W^{\mathrm{sep}}\right]$$

DEVICE DEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{A}_{a|x}\}, \{\overline{B}_{b|y}\}$

Deciding if a behaviour comes from a causally non-sep W: ***SDP***

$$p^Q(ab|\overline{A}_{a|x}, \overline{B}_{b|y}) \neq \mathrm{Tr}\left[(\overline{A}_{a|x} \otimes \overline{B}_{b|y}) W^{\mathrm{sep}}\right]$$

$$\begin{aligned} \text{given} \quad & \{p^Q(ab|xy)\}, \{\overline{A}_{a|x}\}, \{\overline{B}_{b|y}\} \\ \text{find} \quad & W \\ \text{subject to} \quad & p^Q(ab|xy) = \mathrm{Tr}\left[(\overline{A}_{a|x} \otimes \overline{B}_{b|y}) W\right] \ \forall\, a, b, x, y \\ & W \in \mathrm{SEP}, \end{aligned}$$

M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, C. Brukner, New J. Phys. 17, 102001 (2015)

DEVICE DEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{A}_{a|x}\}, \{\overline{B}_{b|y}\}$

*Can **all** causally non-sep process matrices be DD-certified?*

***Yes*.**

DEVICE DEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{A}_{a|x}\}, \{\overline{B}_{b|y}\}$

*Can **all** causally non-sep process matrices be DD-certified?*

***Yes**.*

(tomographically complete instruments)

DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}$

$$p^Q(ab|xy) \neq \mathrm{Tr}\left[(A_{a|x} \otimes B_{b|y})W^{\mathrm{sep}}\right]$$

DEVICE INDEPENDENT

$$\{p^Q(ab|xy)\}$$

$$p^Q(ab|xy) \neq \mathrm{Tr}\left[(A_{a|x} \otimes B_{b|y})W^{\mathrm{sep}}\right]$$

- ***All causal behaviours*** *can be attained by causally separable process matrices and pairs of instruments.*

$$\forall\ \{p^{\mathrm{causal}}(ab|xy)\},\quad \exists\ \{A_{a|x}\},\{B_{b|y}\},\ W^{\mathrm{sep}}\ ;\quad p^{\mathrm{causal}}(ab|xy) = \mathrm{Tr}\left[(A_{a|x} \otimes B_{b|y})W^{\mathrm{sep}}\right]$$

DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}$

$$p^Q(ab|xy) \neq \mathrm{Tr}\left[(A_{a|x} \otimes B_{b|y})W^{\mathrm{sep}}\right]$$

$\Longrightarrow$

A process matrix can be (DI) certified to be ***causally non-separable*** *iff it generates a* ***non-causal behaviour.***

DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}$

Deciding if a behaviour came from a causally non-sep W: ***LIN-PROG***

$$p^Q(ab|xy) \neq \mathrm{Tr}\left[(A_{a|x} \otimes B_{b|y})W^{\mathrm{sep}}\right]$$

$$\begin{aligned} \text{given} \quad & \{p^Q(ab|xy)\} \\ \text{find} \quad & q, \{p^{A\prec B}(ab|xy)\}, \{p^{B\prec A}(ab|xy)\} \\ \text{s.t.} \quad & p^Q(ab|xy) = q\, p^{A\prec B}(ab|xy) + (1-q)p^{B\prec A}(ab|xy) \quad \forall\, a, b, x, y \end{aligned}$$

[3]C. Branciard, M. Araújo, A. Feix, F. Costa, Č. Brukner, New, J, Phys. 18, 013008 (2016)

DEVICE INDEPENDENT

$$\{p^Q(ab|xy)\}$$

Is there a causally non-sep process matrix that can be DI-certified?

Yes.

DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}$

Is there a causally non-sep process matrix that can be DI-certified?

Yes.

W^{OCB} *, violates the GYNI inequality*

O. Oreshkov, F. Costa, C. Brukner, Nat. Comm. 3, 1092 (2012)

DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}$

Can ***all*** *causally non-sep process matrices be DI-certified?*

***No*.**

DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}$

*Can **all** causally non-sep process matrices be DI-certified?*

***No**.*

*There exists causally non-separable process matrices that **cannot** generate non-causal behaviours.*

$$W^{\text{FAB}}$$

'causal' process matrices, do not violate any causal inequality

A. Feix, M. Araújo, Č. Brukner. New J. Phys. 18, 083040 (2016)

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

$$p(ab|x\,,\overline{B}_{b|y}) = \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y})W\right]$$

Cannot be related to the probabilities alone;

cannot be related to the process matrix alone.

- *Need new mathematical object.*

SEMI-DEVICE INDEPENDENT

$$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$$

$$p(ab|x\,,\overline{B}_{b|y}) = \mathrm{Tr}\left[(A_{a|x}\otimes\overline{B}_{b|y})W\right]$$

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

$$p(ab|x\,,\overline{B}_{b|y}) = \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y})W\right]$$

$$= \mathrm{Tr}\left[\overline{B}_{b|y}\mathrm{Tr}_{A_I A_O}(A_{a|x} \otimes \mathbb{I}^B\ W)\right]$$

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

$$p(ab|x\,,\overline{B}_{b|y}) = \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y})W\right]$$

$$= \mathrm{Tr}\left[\overline{B}_{b|y} \underbrace{\mathrm{Tr}_{A_I A_O}(A_{a|x} \otimes \mathbb{I}^B\; W)}\right]$$

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

$$p(ab|x\,, \overline{B}_{b|y}) = \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y})W\right]$$

$$= \mathrm{Tr}\left[\overline{B}_{b|y} \underbrace{\mathrm{Tr}_{A_I A_O}(A_{a|x} \otimes \mathbb{I}^B \; W)}\right]$$

$$= \mathrm{Tr}\left[\overline{B}_{b|y} w^Q_{a|x}\right]$$

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

$$p(ab|x\,,\overline{B}_{b|y}) = \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y})W\right]$$

$$= \mathrm{Tr}\left[\overline{B}_{b|y} \underbrace{\mathrm{Tr}_{A_I A_O}(A_{a|x} \otimes \mathbb{I}^B\ W)}\right]$$

$$= \mathrm{Tr}\left[\overline{B}_{b|y} w^Q_{a|x}\right]$$

Process assemblage

SEMI-DEVICE INDEPENDENT

Causally ordered assemblage:

process matrix

Causally ordered assemblage:

probabilities

SEMI-DEVICE INDEPENDENT

Causally ordered assemblage:
process matrix

$$\{w_{a|x}^{Q,A\prec B}\} : w_{a|x}^{Q,A\prec B} \in L(H^{B_I B_O})$$

$$w_{a|x}^{Q,A\prec B} = \mathrm{Tr}_{A_I A_O}\left[\left(A_{a|x} \otimes \mathbb{I}^B\right) W^{A\prec B}\right]$$

Causally ordered assemblage:
probabilities

SEMI-DEVICE INDEPENDENT

Causally ordered assemblage:
process matrix

$$\{w^{Q,A\prec B}_{a|x}\} : w^{Q,A\prec B}_{a|x} \in L(H^{B_I B_O})$$

$$w^{Q,A\prec B}_{a|x} = \mathrm{Tr}_{A_I A_O}\left[\left(A_{a|x} \otimes \mathbb{I}^B\right) W^{A\prec B}\right]$$

Causally ordered assemblage:
probabilities

Most general set of operators

$$\{w^{A\prec B}_{a|x}\} : w^{A\prec B}_{a|x} \in L(H^{B_I B_O})$$

that takes a set of instruments to a
causally ordered behaviour:

$$p^{A\prec B}(ab|xy) = \mathrm{Tr}\left[B_{b|y} w^{A\prec B}{}_{a|x}\right]$$

SEMI-DEVICE INDEPENDENT

Causally ordered assemblage:

process matrix

$$\{w_{a|x}^{Q,A\prec B}\} : w_{a|x}^{Q,A\prec B} \in L(H^{B_I B_O})$$

$$w_{a|x}^{Q,A\prec B} = \mathrm{Tr}_{A_I A_O}\left[\left(A_{a|x} \otimes \mathbb{I}^B\right) W^{A\prec B}\right]$$

Causally ordered assemblage:

probabilities

$$\{w_{a|x}^{A\prec B}\} : w_{a|x}^{A\prec B} \in L(H^{B_I B_O})$$

$$w_{a|x}^{A\prec B} = {}_{B_O} w_{a|x}^{A\prec B} \quad \forall\, a, x$$

$$\{w_{a|x}^{B\prec A}\} : w_{a|x}^{B\prec A} \in L(H^{B_I B_O})$$

$$\sum_a w_{a|x}^{B\prec A} = \sum_a w_{a|x'}^{B\prec A} \quad \forall\, x, x'$$

SEMI-DEVICE INDEPENDENT

Causally ordered assemblage:
process matrix

$$\{w_{a|x}^{Q,A\prec B}\} : w_{a|x}^{Q,A\prec B} \in L(H^{B_I B_O})$$

$$w_{a|x}^{Q,A\prec B} = \mathrm{Tr}_{A_I A_O}\left[\left(A_{a|x} \otimes \mathbb{I}^B\right) W^{A\prec B}\right]$$

=

Causally ordered assemblage:
probabilities

$$\{w_{a|x}^{A\prec B}\} : w_{a|x}^{A\prec B} \in L(H^{B_I B_O})$$

$$w_{a|x}^{A\prec B} = {}_{B_O}w_{a|x}^{A\prec B} \quad \forall\, a, x$$

$$\{w_{a|x}^{B\prec A}\} : w_{a|x}^{B\prec A} \in L(H^{B_I B_O})$$

$$\sum_a w_{a|x}^{B\prec A} = \sum_a w_{a|x'}^{B\prec A} \quad \forall\, x, x'$$

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

$\Longrightarrow$

*A process matrix can be (SDI) certified to be **non-causally separable** iff it can generate a **non-causal assemblage***

SEMI-DEVICE INDEPENDENT

$$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$$

$$p^Q(ab|x\,,\overline{B}_{b|y}) \neq \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y})W^{\mathrm{sep}}\right]$$

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

Deciding if an assemblage came from a causally non-sep W: ***SDP***

$$p^Q(ab|x\,,\overline{B}_{b|y}) \neq \mathrm{Tr}\left[(A_{a|x} \otimes \overline{B}_{b|y})W^{\mathrm{sep}}\right]$$

$$\begin{aligned}
\text{given} \quad & \{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\} \\
\text{find} \quad & \{w_{a|x}\} \\
\text{s.t.} \quad & p^Q(ab|xy) = \mathrm{Tr}(\overline{B}_{b|y}\, w_{a|x}) \ \forall\, a, b, x, y \\
& \{w_{a|x}\} \in \mathrm{CAUSAL},
\end{aligned}$$

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

Is there *a causally non-sep process matrix that can be SDI-certified?*

Yes.

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

Is there *a causally non-sep process matrix that can be SDI-certified?*

Yes.

DI-certifiable is also SDI-certifiable.

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

*Can **all** causally non-sep process matrices be SDI-certified?*

No.

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

*Can **all** causally non-sep process matrices be SDI-certified?*

***No**.*

*There exists causally non-separable process matrices that **cannot** generate non-causal assemblages.*

$$W \notin \text{SEP}; \; W^{T_A} \in \text{SEP}$$

W^{FAB} : *Not only these process matrices cannot the certified in a DI way, but also not in a SDI way.*

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

Is there *a causally non-sep process matrix that can be SDI-certified but that* ***cannot*** *be DI-certified?*

Yes.

SEMI-DEVICE INDEPENDENT

$\{p^Q(ab|xy)\}, \{\overline{B}_{b|y}\}$

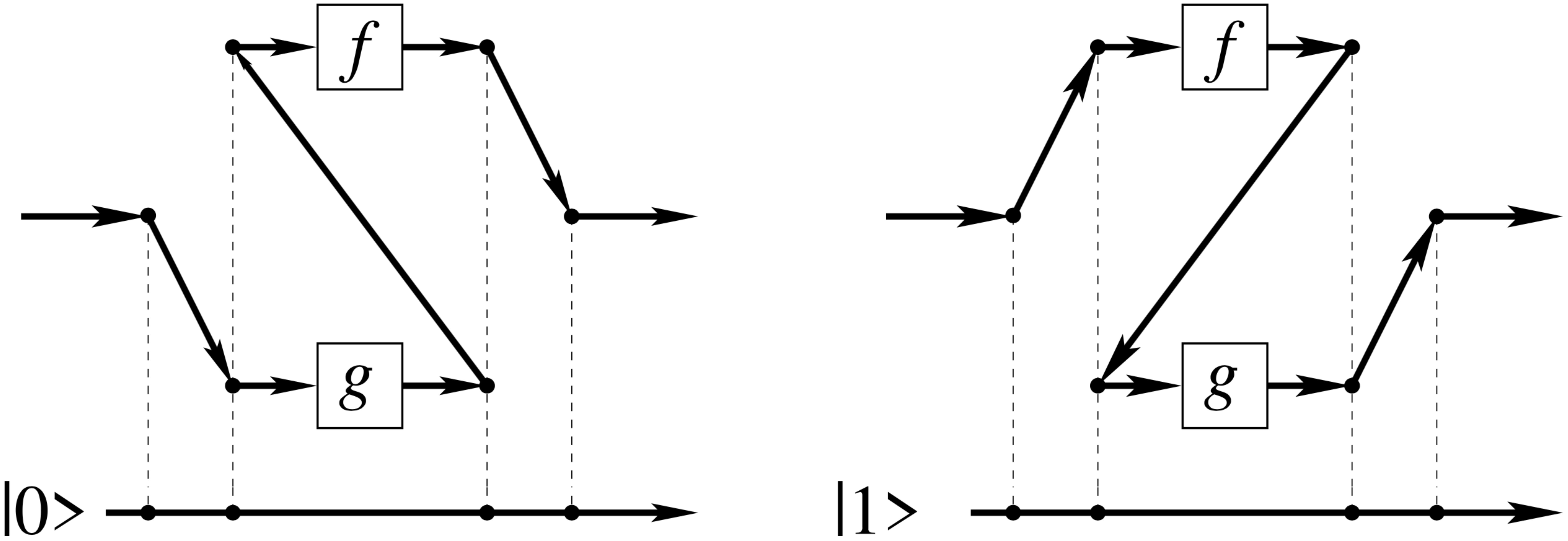
*Is there a causally non-sep process matrix that can be SDI-certified but that **cannot** be DI-certified?*

Yes.

W^{switch}

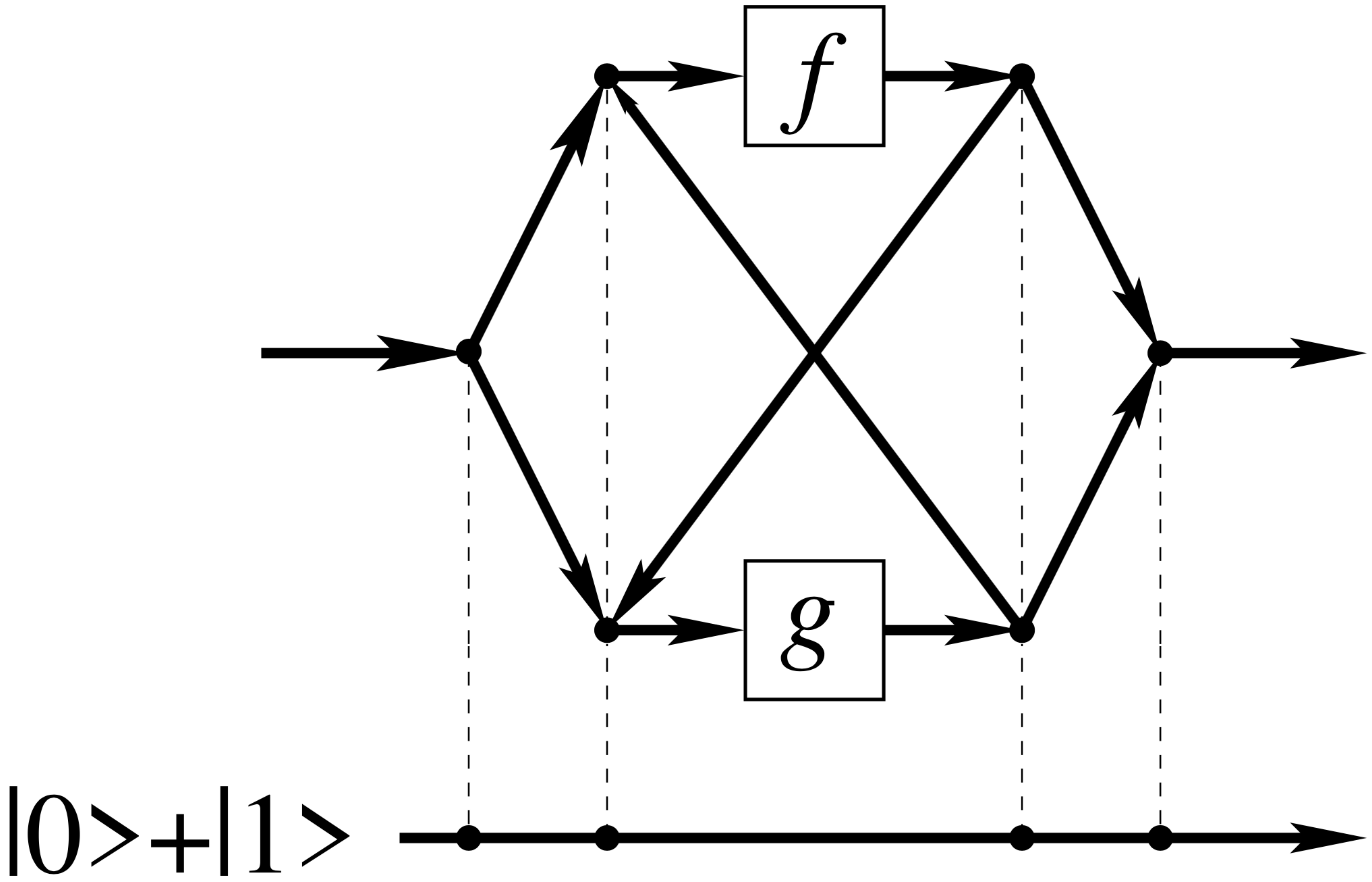
The quantum switch.

THE QUANTUM SWITCH



SOURCE: G. Chiribella, G. M. D'Ariano, P. Perinotti, B. Valiron, Phys. Rev. A 88, 022318 (2013) - FIG. 02

THE QUANTUM SWITCH



SOURCE: G. Chiribella,G. M. D'Ariano, P. Perinotti, B. Valiron, Phys. Rev. A 88, 022318 (2013) - FIG. 03

Device-dependent experiments based on the quantum switch:

- L. M. Procopio, *et al.*, *Nat. Comm.* **6**, 7913 (2015)
- G. Rubino, *et al.*, *Science Advances* **3**, 3 (2017)
- K. Goswami, *et al.*, *PRL* **121**, 090503 (2018)
- G. Rubino, *et al.*, *QIM V: Quantum Tech.*, S3B.3. (2019)
- K. Goswami, *et al.*, arXiv: 1807.07383 (2018)
- M. Taddei, *et al.*, arXiv: 2002.07817 (2020)

THE QUANTUM SWITCH

control

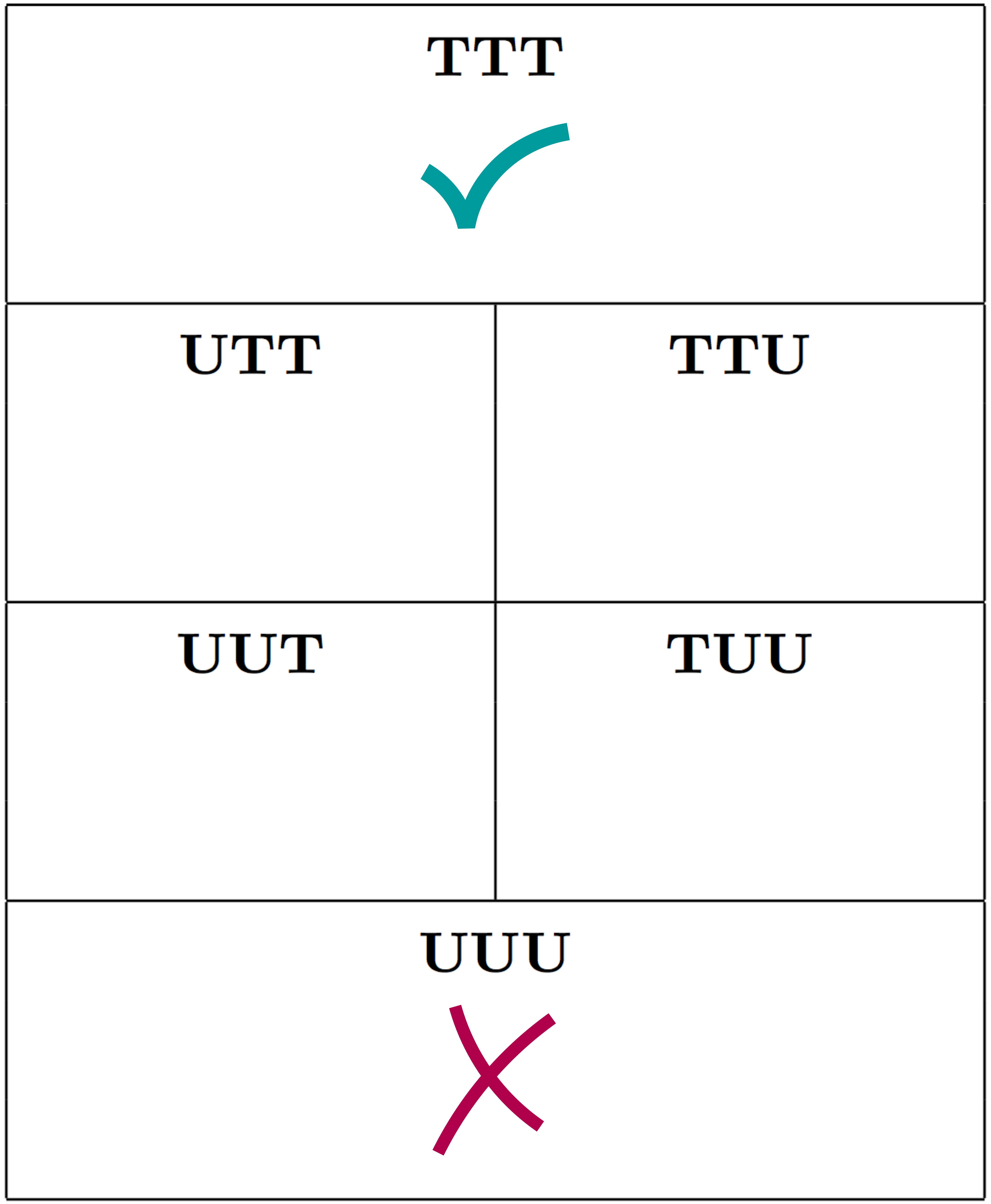
$$W_{\text{switch}} \in L(H^{A_I A_O} \otimes H^{B_I B_O} \otimes H^C)$$

$$W_{\text{switch}} \neq q\, W^{A \prec B \prec C} + (1-q) W^{B \prec A \prec C}$$

	A	B	C
DD	**T**	**T**	**T**
SDI			
SDI			
SDI			
SDI			
DI	**U**	**U**	**U**

	A	B	C
DD	T	T	T
SDI	T	T	U
SDI	T	U	U
SDI	U	T	T
SDI	U	U	T
DI	U	U	U

THE QUANTUM SWITCH



M. Araújo, C. Branciard, F. Costa, A. Feix, C. Giarmatzi, C. Brukner, New J. Phys. 17, 102001 (2015)

C. Branciard, Scientific Reports 6, 26018 (2016)

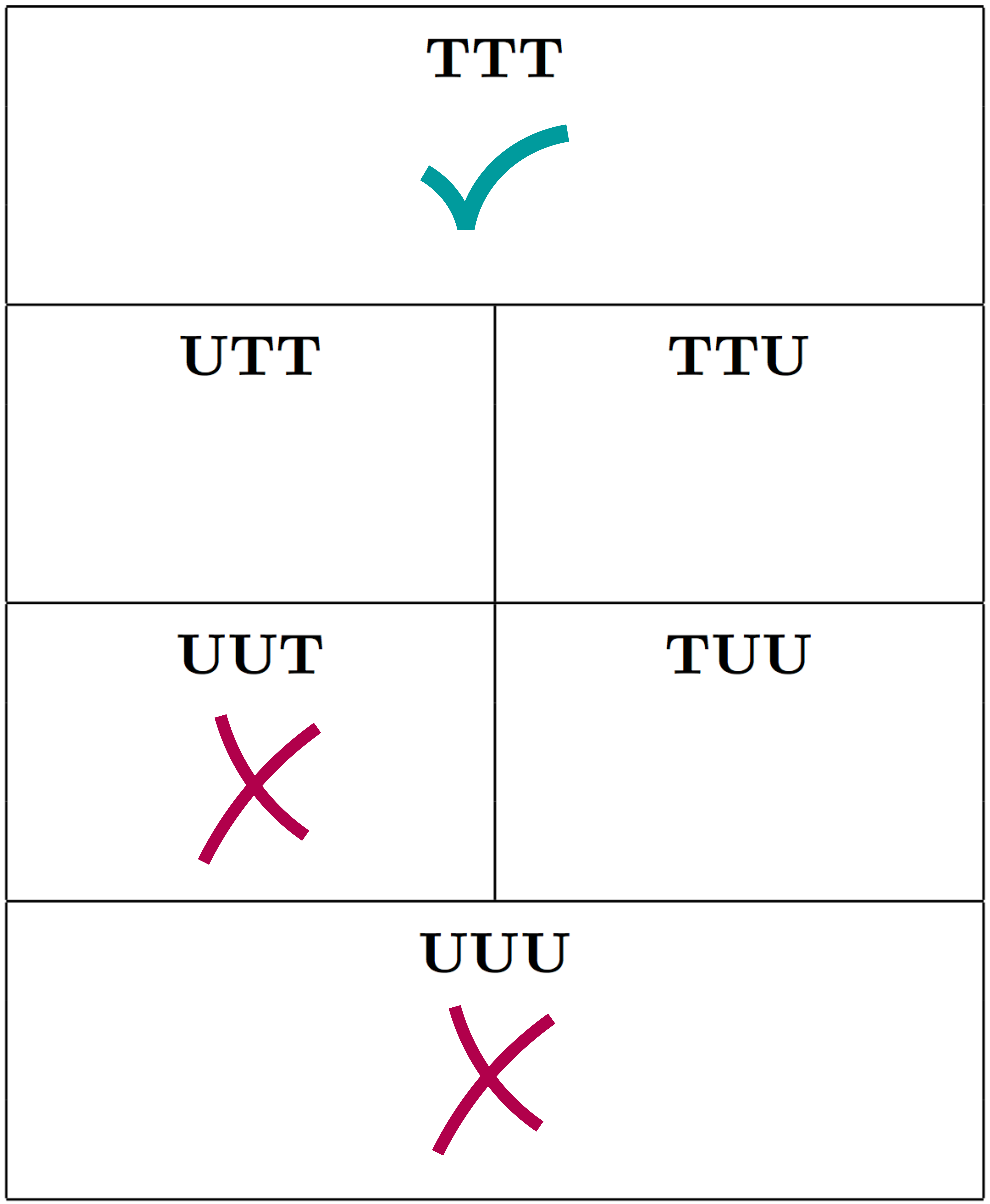
THE QUANTUM SWITCH: UUT

Any tripartite process matrix $W \in L(H^{A_I A_O B_I B_O} \otimes H^C)$ *that satisfies the property*

$$\mathrm{Tr}[(A_{a|x}^{A_I A_O} \otimes B_{b|y}^{B_I B_O} \otimes \mathbb{I}^C) W^{A_I A_O B_I B_O C}] = q\, p^{A \prec B}(ab|xy) + (1-q) p^{B \prec A}(ab|xy),$$

cannot *be certified to be causally non-separable in a UUT scenario.*

THE QUANTUM SWITCH



THE QUANTUM SWITCH: UTT, TTU, AND TUU

The quantum switch ***can be*** *certified in the UTT, TTU, and TUU scenarios.*

THE QUANTUM SWITCH: UTT, TTU, AND TUU

The quantum switch ***can be*** *certified in the UTT, TTU, and TUU scenarios.*

$$A_{0|0}^{A_I A_O} = B_{0|0}^{B_I B_O} = |0\rangle\langle 0| \otimes |0\rangle\langle 0|, \qquad M_{0|0}^{C} = |+\rangle\langle +|,$$
$$A_{1|0}^{A_I A_O} = B_{1|0}^{B_I B_O} = |1\rangle\langle 1| \otimes |1\rangle\langle 1|, \qquad M_{1|0}^{C} = |-\rangle\langle -|,$$
$$A_{0|1}^{A_I A_O} = B_{0|1}^{B_I B_O} = |+\rangle\langle +| \otimes |+\rangle\langle +|,$$
$$A_{1|1}^{A_I A_O} = B_{1|1}^{B_I B_O} = |-\rangle\langle -| \otimes |-\rangle\langle -|,$$

THE QUANTUM SWITCH

TTT ✓	
UTT ✓	TTU ✓
UUT ✗	TUU ✓
UUU ✗	

	DD	SDI	DI
W^{FAB}			
W^{switch}			
W^{OCB}			

	DD	SDI	DI
W^{FAB}	✓		
W^{switch}	✓		
W^{OCB}	✓		

	DD	SDI	DI
W^{FAB}	✓		✗
W^{switch}	✓		✗
W^{OCB}	✓		✓

	DD	SDI	DI
W^{FAB}	✓	✗	✗
W^{switch}	✓	✓	✗
W^{OCB}	✓	✓	✓

CONCLUSION

CONCLUSION

DEVICE DEPENDENT

SEMI-DEVICE INDEPENDENT

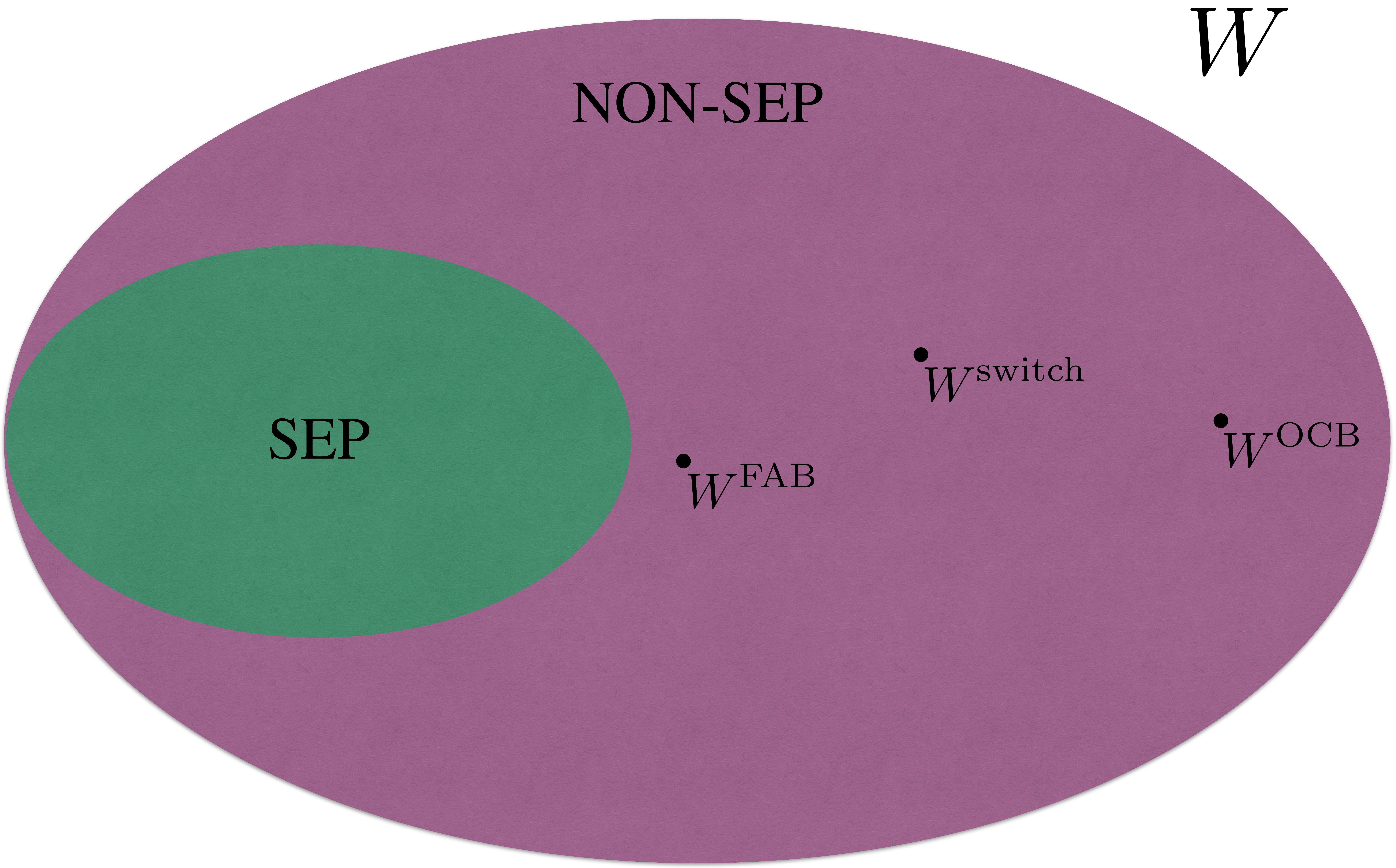
DEVICE INDEPENDENT

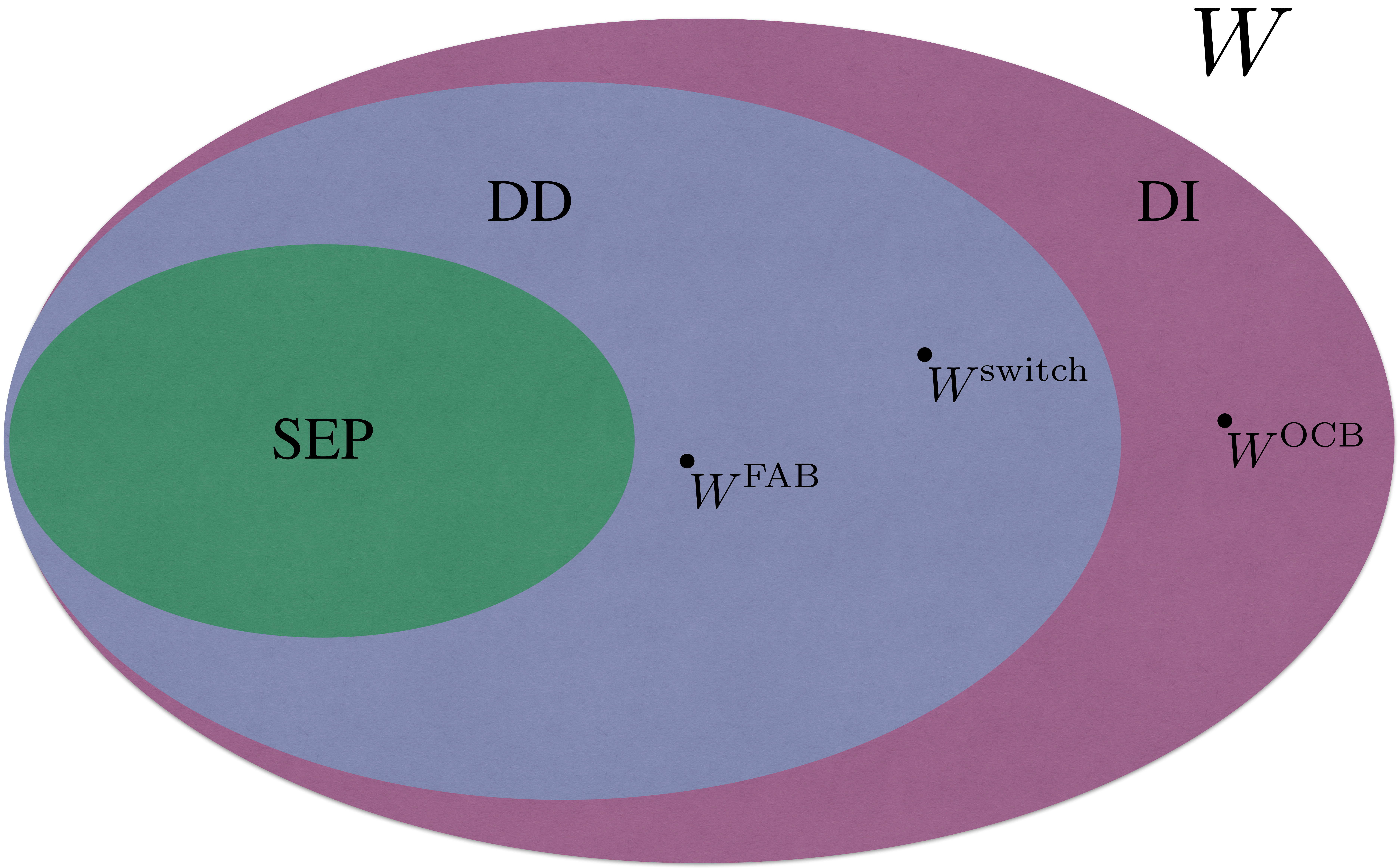
CONCLUSION

DEVICE DEPENDENT	$p^Q(ab\|\overline{A}_{a\|x}, \overline{B}_{b\|y}) \neq \mathrm{Tr}\left[(\overline{A}_{a\|x} \otimes \overline{B}_{b\|y})W^{\mathrm{sep}}\right]$
SEMI-DEVICE INDEPENDENT	$p^Q(ab\|x, \overline{B}_{b\|y}) \neq \mathrm{Tr}\left[(A_{a\|x} \otimes \overline{B}_{b\|y})W^{\mathrm{sep}}\right]$
DEVICE INDEPENDENT	$p^Q(ab\|xy) \neq \mathrm{Tr}\left[(A_{a\|x} \otimes B_{b\|y})W^{\mathrm{sep}}\right]$

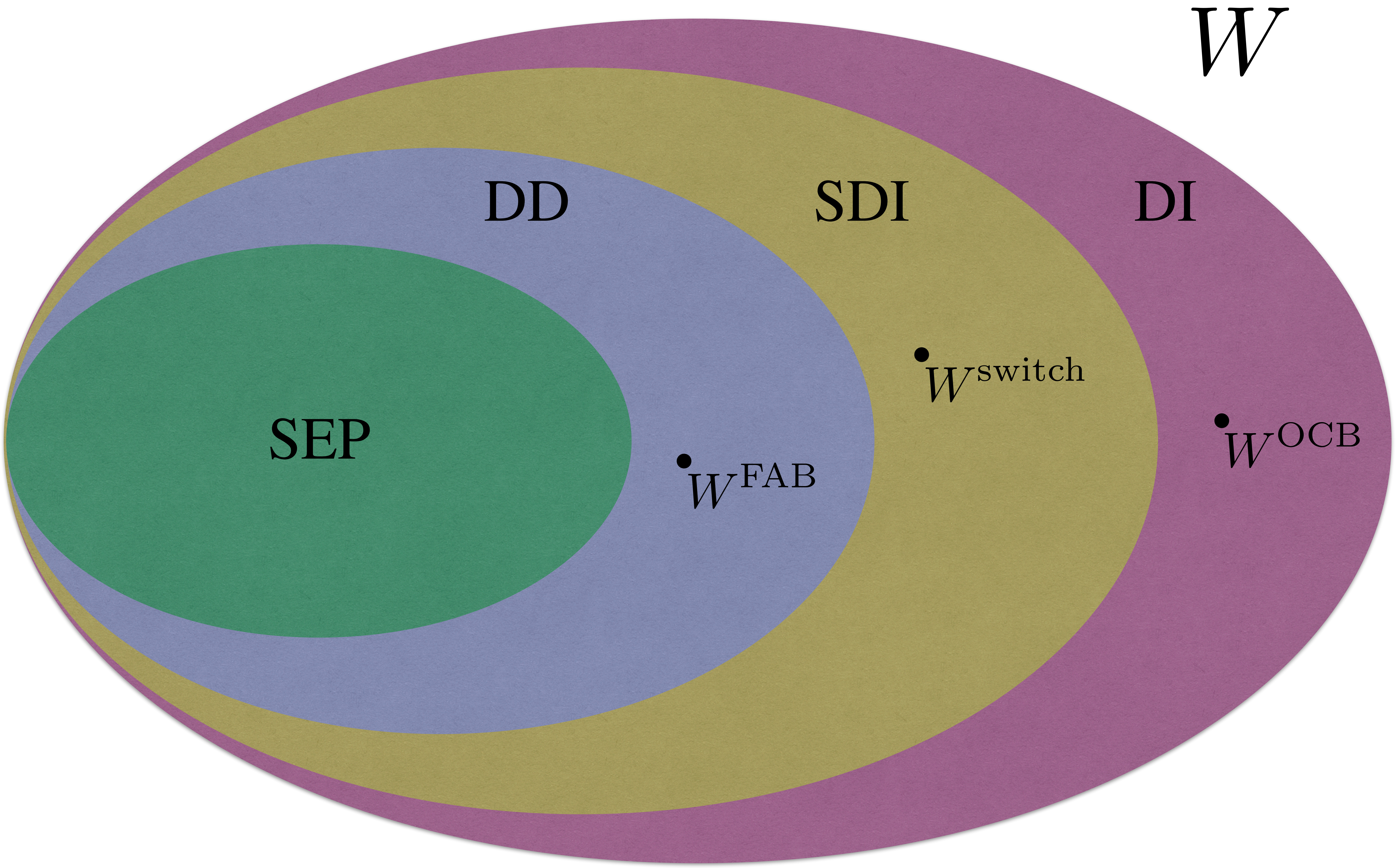
CONCLUSION

DEVICE DEPENDENT	$p^Q(ab\vert\overline{A}_{a\vert x}, \overline{B}_{b\vert y}) \neq \mathrm{Tr}\left[(\overline{A}_{a\vert x} \otimes \overline{B}_{b\vert y})W^{\mathrm{sep}}\right]$	$p^Q(ab\vert\overline{A}_{a\vert x}, \overline{B}_{b\vert y}) \neq \mathrm{Tr}\left[(\overline{A}_{a\vert x} \otimes \overline{B}_{b\vert y})W^{\mathrm{sep}}\right]$
SEMI-DEVICE INDEPENDENT	$p^Q(ab\vert x, \overline{B}_{b\vert y}) \neq \mathrm{Tr}\left[(A_{a\vert x} \otimes \overline{B}_{b\vert y})W^{\mathrm{sep}}\right]$	$p^Q(ab\vert x, \overline{B}_{b\vert y}) \neq \mathrm{Tr}\left[\overline{B}_{b\vert y} w^{\mathrm{causal}}_{a\vert x}\right]$
DEVICE INDEPENDENT	$p^Q(ab\vert xy) \neq \mathrm{Tr}\left[(A_{a\vert x} \otimes B_{b\vert y})W^{\mathrm{sep}}\right]$	$p^Q(ab\vert xy) \neq p^{\mathrm{causal}}(ab\vert xy)$


W
NON-SEP
SEP
W^{switch}
W^{OCB}
W^{FAB}



W
DD
DI
SEP
W^{switch}
W^{OCB}
W^{FAB}



W
DD
SDI
DI
SEP
W^FAB
W^switch
W^OCB

Thank you.